TRANSMISSION PRICING IN DEREGULATED ENVIRONMENT

By

Major Rajesh Vaid

DEPARTMENT OF ELECTRICAL ENGINEERING

Indian Institute of Technology Kanpur

JANUARY, 2002

Transmission Pricing in Deregulated Environment

A Thesis Submitted

in Partial Fulfilment of the Requirements

for the Degree of

MASTER OF TECHNOLOGY

by

Major Rajesh Vaid

to the

DEPARTMENT OF ELECTRICAL ENGINEERING & ACES

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

January, 2002

CERTIFICATE

It is certified that the work contained in the thesis entitled " Transmission Pricing in Deregualted Environment", by 'Major Rajesh Vaid' , has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

January, 2002.

Dr. P.K. Kalra
Professor
Department of Electrical Engineering,
Indian Institute of Technology,
Kanpur.

Acknowledgement

I express my deep gratitude to Dr. P. K. Kalra for the inspiration, invaluable guidance and supervision. He was always at hand with inspiring ideas and motivation, for clearing the doubts and showing the path further.

I am also thankful to him for providing excellent atmosphere to work in the lab, besides allowing for free and frank discussion and exchange of views on the subject.

I would also like to thank Dr. Puneet Chitkara who spared invaluable time and provided guidance in relating to the economical aspects of the work. He was a source of constant help and motivation during the entire period.

My thanks are also due to Major Jayesh Adhvaryu for constantly supporting me as well as to Major Mohan, Sqn. Ldr P A Patil, Saleem, Bhavani Shankar, Manoj, Madhav Krishna and Prasanth for maintaining a cordial and healthy working environment in the Lab. My thanks also go to all my instructors at IIT, Kanpur for imparting knowledge on the subject of Power Systems to me.

I am also grateful to the Indian Army for giving me an opportunity to undergo this course at this esteemed institution.

Last but not the least; I acknowledge the whole hearted support my wife Geeta, sons Raul and Rijul gave me during the M. Tech course. They were always there in the times of need and put up with my "irregular timings" with smiling faces.

ABSTRACT

Recent times have seen many countries both developed and developing open their traditional sectors to the market forces and competition. Electrical power industry was also one such traditional sector, which till a few years back was a fully regulated industry. It was a vertically integrated industry where a utility operated generation, transmission, distribution and provision of services to the consumers. The consumer had no choice and was used to buying a bundled commodity – electricity. Due to various reasons ranging from creating competition and more efficient operations in the developing countries; to problems of lack of resources and lack of return on the investment already in place for the developing countries, this sector has seen deregulation or re-regulation take place. It is because of this that pricing mechanisms too are undergoing change in accordance with the change in the industry from a single part tariff to two part tariff..

In this thesis, the variable component of the two part tariff resulting from the use of energy on the part of the energy supply utility has been studied. The emphasis has been on the pricing of reactive power, optimal placement of reactive support at the load bus and calculating the required optimal value of this capacitance.

CONTENTS

List of Tables

CHAPTER 1

INTRODUCTION

1.1 GENERAL

The electric power industry in all the major countries and in a few developing countries has undergone a major change from the way it used to be operated some years back. The power industry which till a recent past was operated an integrated monopoly. The same company (utility) owned generation, transmission and distribution. In the restructuring that was carried out in the power sector, by the way of de-regulation (or re-regulation) the utilities were broken down in to separate entities. In the developed countries the aim behind it was to provide the consumer with cheaper energy and to instill more efficient operations, where as in the developing nations this was resorted to as a means of attracting investment for the growth of the sector and also to better the performance of the existing utilities.

The transmission being an important link in the process of supplying power to the consumer and because of the typical nature of this segment it has been debated for long. Unlike the generation and distribution sectors, in the re-regulated environment transmission sector was to be operated as a regulated monopoly. The company owning the transmission lines (wires) had no competition in its field, largely due to the behavior of the electricity. Moreover, for the consumer to get cheaper power it was important that the transmission industry should have such tariffs which allows the generators and the distribution companies to interact better and at the same time makes most efficient use of the assets of transmission. Hence, the economics related to tariffs for transmission of power assume importance and are therefore under continuous development.

Further, since transmission is complement to generation to some extent, the tariffs charged by transmission industry should be allocated fairly to the user of this asset. Tariffs charged arbitrarily can lead to wrong signals being sent to the

consumers and the investors. This can further lead to improper development of the both generation resources as also in the pattern of the consumption of power. There are various means of calculating tariffs that have been dealt in detail in the literature, especially by Phillipson and Willis [10].

Tariffs for transmission in our country are charged on flat fee basis, thus making no distinction between the consumers. Also as tariff does not have any distinction on the part of variable cost and fixed costs. It has been debated over long that such tariff structures should come in place.

During the recent times a lot of pioneering work has been done in this area dealing with de-regulation. Some of the transmission related issues have been discussed by Richard Christie et al. in Ref [1]. The authors use the DC formulation to arrive at nodal costs. The issues relating to optimal reactive support and the pricing of reactive support have been dealt by Kankar Bhattacharya et al. in ref [2], [3], also they have suggest a method to calculate the optimal placement and sizing of capacitors. Reactive power as an ancillary has been discussed in detail by J Fu et al. in Ref [4] which includes the biding process.

1.2 ORGANIZATION OF THESIS WORK

1.2.1 Aim of the thesis: The aim of the thesis is to study transmission system related issues with an emphasis on pricing of transmission of energy.

1.2.2 Scope of the thesis: The thesis used load flow solution and the optimal power flow techniques to determine the price for the transmission of power. The formulation of load flow and AC OPF were done using the GAMS software. The problem was studied by using various price curves. To arrive at the cost of transmission reactive power was taken as function of sinh x and inverse tanh x. Also, effect of static reactive support devices like capacitors and transformer with variable taps was studied. The scope of the thesis is restricted to the steady state operation of power system.

1.2.3 The thesis is organized in the following chapters:

Chapter 2 relates to the issues of transmission and pricing mechanisms. Besides an overview, it includes a short description on different pricing methods, why is pricing for transmission required. This also gives out the objectives that should accrue from pricing of transmission as well as the signals sent out by it.

Chapter 3 gives out the background of Newton Raphson technique of load flow. It also gives in general the formulation of optimal power flow (OPF) used for the solving problem, besides it also gives out various techniques that are used for solving AC OPF.

Chapter 4 relates to the teat cases and the results as obtained after studying various systems. It gives out comparison between various results obtained and a brief analysis of the results.

Chapter 5 gives out the conclusions and scope for further work.

CHAPTER 2

PRICING MECHANISMS - TRANSMISSION AND ANCILLARY SERVICES

2.1 GENERAL

In the traditional world of electricity regulation, vertically integrated companies providing generation, transmission and distribution services, have supplied power. Consumers have purchased a bundled commodity -- delivered electricity -- and there has been no need to price the components individually. This will not be the case as the electricity market becomes more competitive. A beginning in this respect has been made in our country too, with the setting up of Power Grid Corporation of India (PGCIL).

The transmission lines earlier owned by NTPC and other utilities have been passed on to this corporation and this had been made responsible for the transmission of power at the regional level. Besides, PGCIL another company by the name of Power Trading Corporation too has been set up to deal with the market mechanism that would come up as a result of de-regulation and unbundling of services, taking place in the power sector.

Due to de-regulation and unbundling of the services, it is important that a suitable pricing mechanism be there which helps service providers be it, real power, reactive power, load following or transmission of energy.

2.2 DEREGULATION AND UNBUNDLING OF SERVICES

From the times when the power industry was set up it has been operating as a regulated industry, with utility having a monopoly. They being the only company or government agency that produced, transmitted, distributed and sold electric power and energy services. Mostly, these utilities were state owned. The consumer had no choice, but to buy power from this utility alone. Over the last few years there has been a change in the electric power industry, as it too has been touched by the forces of market mechanisms. Some thought went into it and it

was presumed that competition in this field too could lead to significant improvements in the way this industry is operated. Also, it could encourage investments in the entire range of power industry which may help in bridging the gap between the demand and supply.

Thus various concepts evolved for the industry including deregulation. In order to know what deregulation is, it is must to know the meaning of regulation or regulated industry.

Regulation means that the government has set down certain rules and laws for the industry to operate. It provides industry an environment to do business under fair or fully disclosed practices and operate their facilities under laid down guidelines. Besides, deregulation other concepts evolved ranging from:

- Competition,
- De-Regulation,
- Restructuring,
- Privatization and
- Open Access.

These can be briefly explained as follows:

2.2.1 Competition. Two or more players vying for the same business or opportunity. In the case of power industry this is being created in the wholesale (generation) and retail (distribution) markets. But, specific in our country as we are way short of generation, competition in this field may take a long time to come.

2.2.2 De-Regulation. Government brings about changes in the rules governing the sector so as to encourage competition. The monopoly franchise rules and other rules are modified that effect the operation and business of electric utilities. The customers also have some freedom on how to buy power and electric services.

2.2.3 Re-structuring. Disassembly of electric utility and re-assembly into another form or functional organizations. It often involves the creation of new government agencies or any other type of entity.

2.2.4 Privatization. Sale of state owned electric utility assets and operating company to private company.

2.2.5 Open Access. Government encourages competition in the electric industry. The electric grid (grid) is akin to road-way system connecting all generators and consumers. It is "opened" to use by anyone, rather than the utility. It provides a way for competitive generators and buyers to do business.

Hence, as a result of de-regulation or re-regulation as it is sometimes referred to, some important players also came into being. These were independent system operator (ISO), power exchange (PX) and the regulator. They are as important to the system as the generating company (GENCO), transmission company (TRANSCO) and distribution company (DISCO).

Unbundling of services on the other hand involves ancillary services that are required for the movement of power. Each of the different services is identified as distinct functions or tasks. The provider of these services must price each individually and offer it for sale as separate item.

2.3 TYPES OF PRICING MODELS

2.3.1 *Postage Stamp*

Postage stamp rates have been the most common transmission pricing approach. A postage stamp rate is a flat per kW charge for network access within a particular zone, based on average system costs. The cost for transmitting within the zone is independent of the transmission distance. Postage stamps are the most common type of transmission tariff.

A generator transmitting to a load in a different zone would have to pay the postage stamp charges for the zone of origin and the zone of

delivery, and also any intervening zones. This accumulation of zone access charges is often called "pancaking." Although transmission within a zone is independent of distance, longer distances increase the likelihood that more than one zone will be crossed, which would increase the total transmission cost.

Although postage stamp rates provide a way to recover the fixed costs of the network, they provide no information about congestion when used by themselves.

2.3.2 *Megawatt Mile*

Other types of rates explicitly reflect the fact that the cost of transmission depends on the distance the power is transmitted and how much power is transmitted. Traditional contract path pricing identifies a particular transmission path over which the transmitted power *could* flow. Such rates do not price transmission accurately because they fail to take into account the total effects of the transaction on the network. Flow-based pricing schemes, such as megawatt-mile pricing, were developed as an alternative to contract-path pricing.

Megawatt mile pricing generally involves using load flow analysis to model the power flows on the transmission network to determine transmission distance. These distances, if computed correctly, more accurately reflect the impact of a transmission agreement on the system. The cost of transmission per megawatt-mile is the total cost averaged over megawatt-miles of usage.

Used alone, MW-mile rates provide no information about congestion.

2.3.3 *Contract Path*

Contract path pricing calls for transmission from point A to point B based on the cost of single identified path. The price usually includes a capacity charge to cover the capital costs, and energy charges based on losses and other operating costs.

Suppose two parties want to move 400 MW of power between two points, and choose a transmission line as "contract path" between the two points. Then even knowing that the power will actually move on different parallel paths between the two stations, they calculate the costs to be paid based on this "contract path".

2.3.4 *Locationally Based Marginal Costing (LBMP) or Nodal Pricing*

It is the most complicated, but accurate pricing method. It determines transmission prices at each bus in the system, based on costs including the price of power as measures in the spot market, and the cost of moving power to various locations. Transmission cost for any new transactions taking place between any two buses is then taken as the difference of these costs.

2.4 WHY HAVE TRANSMISSION PRICES?

Two different points of view on this issue can be adopted. Seen as an industry in itself, users of the transmission grid should pay for the use of transmission services and for the provision of investment in transmission technologies. The traditional theory of marginal-cost pricing in microeconomics finds its counterpart in nodal pricing in the power industry.

The transmission industry, being the link between generators and loads, should ensure the reliability of the transmission grid by coordinating efficiently the activity of grid users by introducing prices as coordinating (control) variables.

These two roles of transmission prices are consistent. We can add to them a third objective of transmission pricing; i.e. recovering the total cost of

investment. This condition is essential to the sustainability of any for-profit company.

An investment in transmission capacity has an economic value. It enables to import inexpensive power from low-cost regions to high-cost regions. The new industry structure should then be able to invest in transmission capacity, in order to provide the transmission services users, what they paid for. More than that, the notion of optimal investment is essential. Where to invest, how much capacity and when to invest are three aspects of the same issue.

The transmission industry should be structured in a way that takes into account such aspects of the problem. It is important to note that transmission capacity is a complement to generation capacity. As such, an important aspect of optimal investment consists in coordinating the investment policies in generation and transmission.

Because the principal legacy of the existing regulatory system is an inefficient and costly stock of generating capital, the restructuring discussion has focused on introducing competition and efficient pricing in the generation sector. For transmission, many current proposals emphasize the importance of non-discriminatory open access rather than efficient pricing. If, however, one of the principal goals of restructuring is rationalizing the power industry's capital stock, then the pricing of transmission is also of critical importance, because the relationship between generation and transmission is a close one.

Generation and transmission are complements, which together produce delivered electricity. But, they are also, increasingly, substitutes and this trend is likely to accelerate as competition develops.

Transmission of electricity occurs because of regional differences in generating costs. Efficient pricing of transmission means that prices should equal marginal costs, which consist of marginal line losses and, in the event of a transmission constraint, transmission congestion rents.

Efficient transmission pricing is necessary to assure economic dispatch of existing generation capacity -- delivery of electricity to consumers at its minimum

cost, including both generation and transmission costs. If transmission is priced below its marginal cost, it becomes profitable to import power from distant generating sources, even though less-distant sources might entail a lower (social) cost. If, on the other hand, transmission is priced too high, the geographic market for electricity is artificially narrowed, which limits the importation of low-cost generation and the competitiveness of the generation market in general.

Over time, efficient pricing of transmission is necessary to send the correct signals to the market concerning expansion of existing transmission capacity and the location of new generation capacity. If transmission prices do not accurately reflect the costs of transmission, including transmission constraints, participants in the market will not be able to correctly determine whether those constraints are best addressed through expansion of transmission capacity or the installation of new generation capacity closer to the load.

2.5 POLICY OBJECTIVES FOR TRANSMISSION PRICING

Transmission pricing must send the right price signals to market participants and compensate transmission owners and investors. Specifically, transmission pricing should be able to guide the market such that it:

a) Promotes efficient day-to-day operation and maintenance of the grid *(i.e.,* signal to the *transmission operator*)
b) Signals locational advantages for investment in generation and demand *(i.e.,* signal to *generators* and *customers*)
c) Signals need for investment in the transmissions system *(i.e.,* signal to *transmission investors*)
d) Facilitates economic interconnection of new generators *(i.e.,* another signal to *generators*)
e) Compensates owners of existing transmission assets *(i.e.,* signal to *transmission owners*)
f) Be simple and transparent (important to market participants and public policymakers)

g) Be politically "implementable" (especially important to regulators and public policymakers)

Transmission pricing was (relatively) simple when the transmission operator, owner, investor and generator were all the same entity. Today, however, ISOs operate the system, current owners and future investors diverge, and generators are either divested entirely or managed out of separate affiliates. Price signals send critical information to all users of the electricity system. Accurate price signals are necessary,

• to elicit the demand response from customers needed for efficient competitive markets;

• to indicate the need and *opportunity* for new generation (*through location based marginal pricing*);

• to provide important information to new market entrants about opportunities to offer value-added services to customers (*e.g.*, energy efficiency, distributed generation and new technologies); and

• to identify the need and *opportunity* for new transmission investment.

This last signal is particularly the case for merchant transmission, which some propose as the future of economic grid expansion. Also the last signal is also important under regulation (likely to continue for some time) where old incentives to build transmission (tied to generation) are diminishing if not fully gone.

2.6 PRICING OF REACTIVE SUPPORT AND TRANSFORMER TAPS

Transmission services not only include the movement of real power from one place to other but also the necessary reactive power that may be required for the system. Moreover, the reactive power is required in the system for the balancing of the system and to keep it in a stable state. Further, if there is not adequate reactive support in the system then the losses in the system increase and

useful energy is wasted in pushing power. This can be set right by proper reactive support through devices like SVC and capacitors. These devices help in maintaining the system voltages and hence reducing the loss of energy.

Besides, these devices voltages in the line can also be controlled by use of transformer taps. These tap changers do have a small range in which they are effective. The range is generally + or – 10%. Since changing of the taps causes wear and tear on the transformer, and has adverse effect on the equipment.

Hence for efficient running of the transmission system some such devices are required. As capital cost is incurred in procurement and installation of these systems it is but natural that the operator must have some method to recover this component of the cost too. In this work, a method has been suggest to price these components (i.e. tap changing and reactive support by way of capacitors), which has been described in chapter 4. Also, the same methodology has then been use to calculate the cost for transmission.

CHAPTER 3

POWER FLOW AND OPTIMAL POWER FLOW

3.1 INTRODUCTION

The ordinary power flow or load flow problem is stated by specifying the loads in megawatts (MW) and megavars (MVARs) to be supplied at certain nodes or busbars (bus) of a transmission system and by the generated powers and the voltage magnitudes at the remaining nodes of this system together with a complete topological description of the system including its impedances. The objective is to determine the complex nodal voltages from which all other quantities like line flows, currents and losses can be derived. In mathematical terms the problem can be reduced to a set of nonlinear equations where the real and imaginary components of the nodal voltages are the variables. The number of equations equals twice the number of nodes. The nonlinearities can roughly be classified being of a quadratic nature.

The load flow solution is a solution of the network under steady state condition subjected to certain inequality constraints under which the system operates. These constraints can be in the form of load nodal voltages, reactive power generation of the generators, the tap settings of a tap changing under load transformer etc.

Thus, load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting transmission lines. Load flow solution is essential for designing a new power system and for planning extension of the existing one for increased load demand. This analysis requires calculation of numerous load flows under both normal and abnormal (outage of transmission line, or outage of some generating source) operating conditions. Load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied.

In short it can be stated that, the result of a power flow problem tells the operator or a planner of a system in which way the lines in the system are loaded, what the voltages at the various buses are, how much of the generated power is lost and where limits are exceeded.

Single line representation is enough under balanced operating conditions. A load flow solution of the power system requires mainly the following steps:

- Formulation of the network equations.
- Suitable mathematical technique for solution of the equations.

Under steady state condition the network equations will be in the form of simple algebraic equations. The load and hence generation are continually changing in a real power system, but for solving load flow it is assumed that loads and hence generation are fixed at a particular value over a suitable period of time. E.g. say, half an hour or so. In that, load flow solution gives a snap shot of the transmission at a particular instance.

In a power system each bus or node is associated with four quantities, real and reactive powers, bus voltages magnitude and its phase angle. In a load flow solution two of the four quantities are specified and the remaining two are required to be ascertained through the solution of these equations. The buses are classified depending upon the quantities specified into the following three categories:

3.1.1 **Load bus:** At this bus the real and reactive components of power are specified. It is desired to find out the voltage magnitude V and phase angle δ through the load flow solution.

3.1.2 **Generator bus or voltage controlled bus:**

Here the voltage magnitude corresponding to the generation voltage and real power P_G corresponding to its ratings is specified. It is required to find out the reactive power generation Q_G and the phase angle δ of the bus voltage.

3.1.3 Slack, swing or reference bus:

Here the voltage, magnitude V and phase angle δ are specified. This will take care of the additional power generation required and Transmission losses. It is required to find of real and reactive power generations (P_G, Q_G) at this bus.

3.2 NEWTON-RAPHSON LOAD FLOW (NRLF) METHOD

For an N-bus power system there will be n-equations for real power P_i and n-equation for reactive power Q_i.

$$P_i = P_{Gi} - P_{Di} = \sum_{j=1}^{n} \left[V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \right]$$

$$Q_i = Q_{Gi} - Q_{Di} = \sum_{j=1}^{n} \left[V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \right] \quad \ldots \text{(3.1)}$$

$$i = 1,2,3,\ldots\ldots,n$$

The number of equations to be solved depends upon the specifications we have. If the total number of buses is n and number of generator buses is m then the number equation to be solved will be number of known P_i's and number of known Q_i's. In the above conditions number of known P_i's are n-1 and the number of known Q_i's are n-m, therefore the total number of simultaneous equation will be 2*n-m-1, and number of unknown quantities also 2*n-m-1. Unknowns to be found out are δ at all the buses except slack (i.e. n-1) and V at load bus (i.e.n-m). Following the method explained in Ref. [7] problem can be formulated as follows:

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{pmatrix} \begin{pmatrix} \Delta \delta \\ \Delta V \end{pmatrix} \quad \ldots \text{(3.2)}$$

Real power terms will be calculated for all the buses except slack and reactive power terms will be calculated for load buses. In the above equation

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} \text{ is the mismatch vector}$$

$$\begin{pmatrix} \Delta \delta \\ \Delta V \end{pmatrix} \text{ is the correction vector}$$

and

$$J = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{pmatrix} \text{ is the Jacobian matrix} \qquad \dots \ (3.3)$$

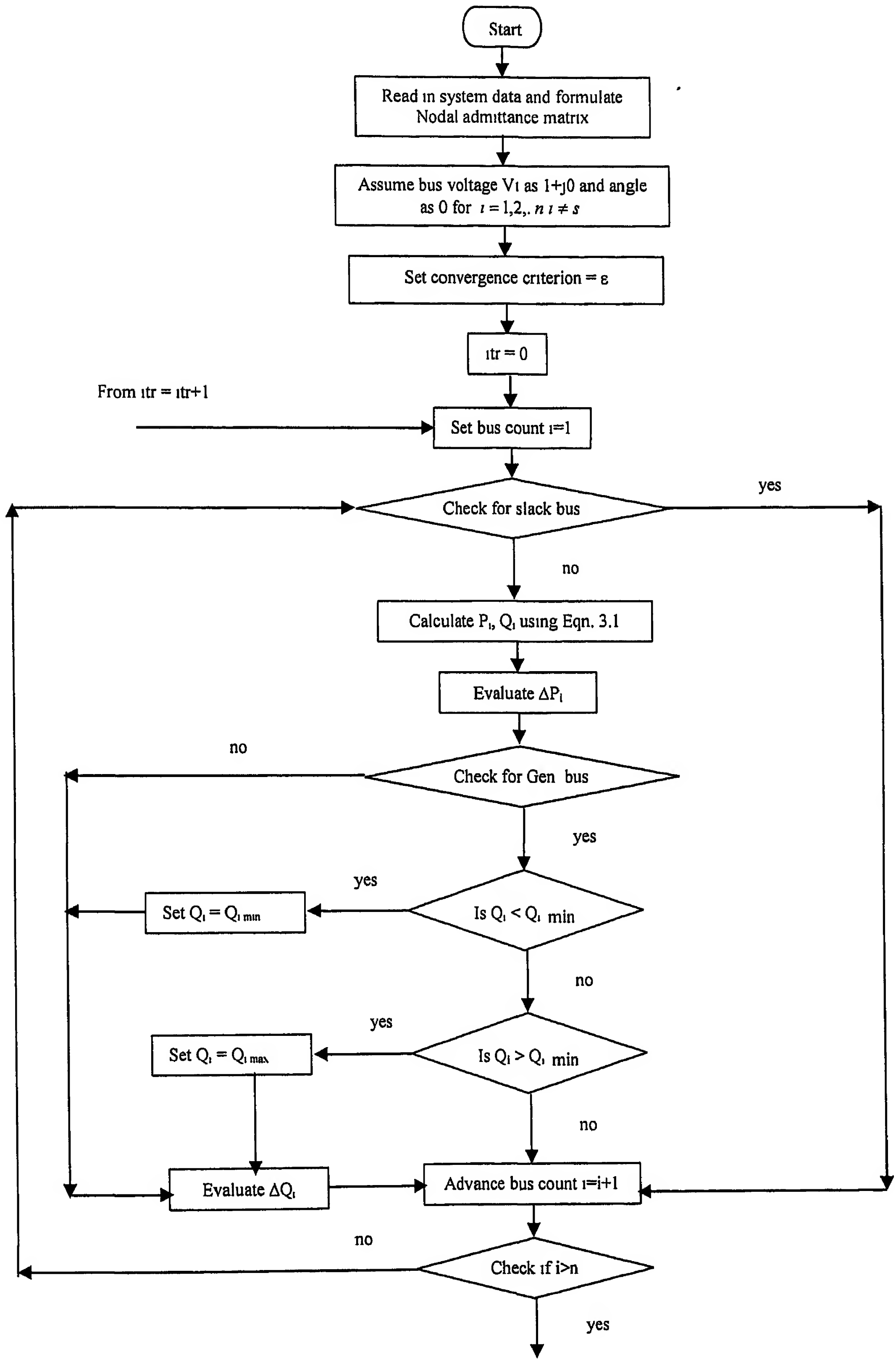
Some constraints are applied on Load flow solution. The solution has to have some limits such as voltage, and reactive power should not be greater/lesser than its limits i.e.

$$Q_{i\,\min} < Q_i < Q_{i\,\max} \qquad \dots (3.4)$$

Voltage has to satisfy the conditioned given below

$$V_{i\,\min} < V_i < V_{i\,\max} \qquad \dots \ (3.5)$$

3.3 FLOW CHART



Continued in the next page

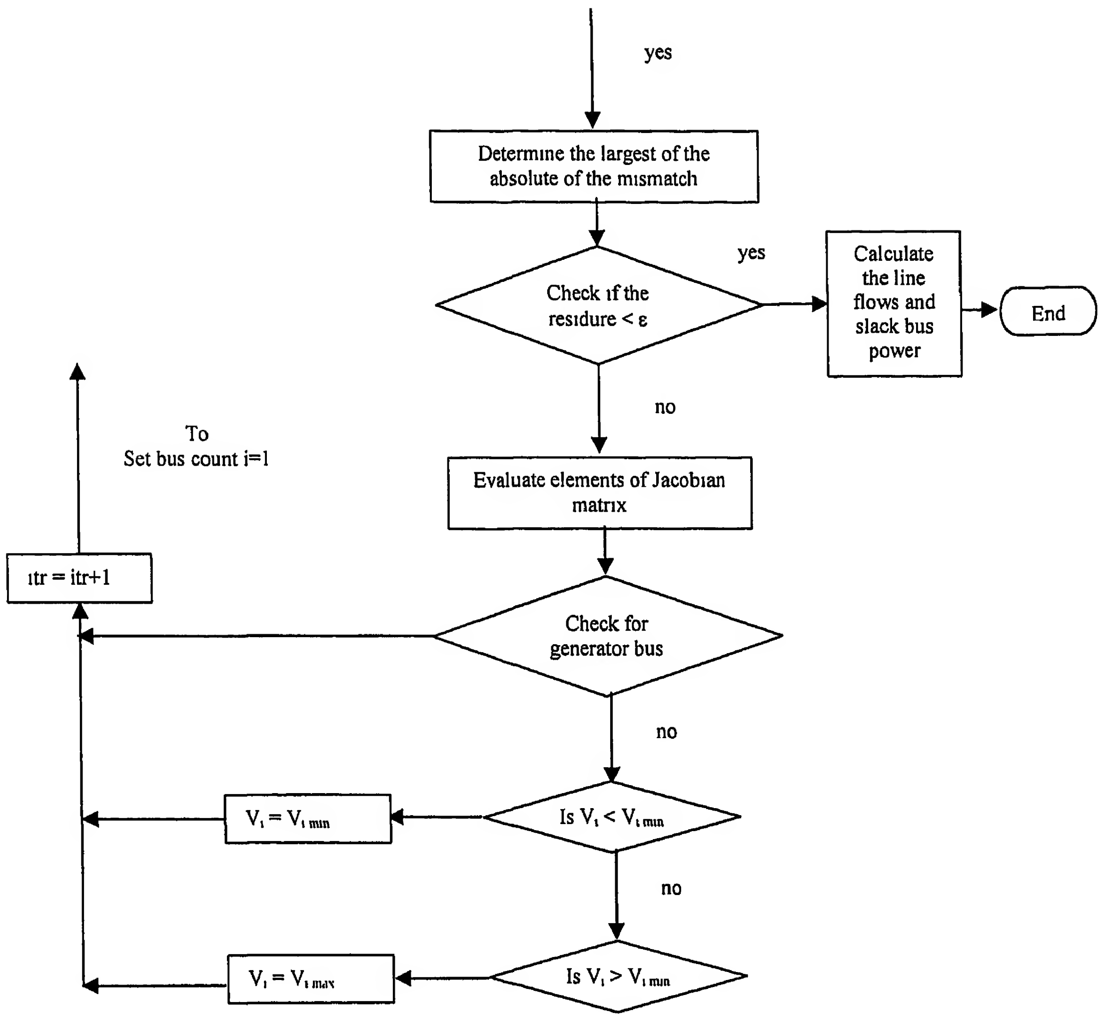


Fig 3.1 Flow chart for load flow solution using Newton-Raphson method

OPTIMAL POWER FLOW

3.4 INTRODUCTION

For the planners and operators load flow solution corresponds to a snapshot only. Planning and operating requirements very often ask for an adjustment of the generated powers according to certain criteria. One such very common operating criterion is to find the minimum of the generating costs. The application of such a criterion immediately suggests variable input powers and bus voltages, which need to be determined so that a minimum of cost of generating these powers is achieved.

At this point it is not only the voltages at nodes where the loads are supplied but also the input powers together with the corresponding voltages at the generator nodes, which have to be determined. The degree of freedom for the choice of inputs seems to be exceedingly large, but due to the presence of an objective, namely to reach the minimum of the generating costs the problem is well defined and, the aim still being the same, i.e. the determination of the nodal voltages in the system. They play the role of state variables from which all other quantities can be derived.

As long as the power flow model stays the same it is considered the optimal power flow problem where the objective is a scalar function of the state variables. In essence, any optimal power flow problem can be reduced to such a form.

But, practical requirements ask for a more realistic definition, the main addition being the statement of constraints. Fortunately, the theory developed by Kuhn and Tucker is able to provide the optimality conditions, which guarantee the correctness of the result in the end. However, the optimality conditions do not offer a solution method.

3.5 VARIOUS OBJECTIVE FUNCTIONS

3.5.1 Cost Minimization Function

The major component of generator operating cost is the fuel input/hour, while maintenance contributes only to a small extent. The input-output curve of a unit can be expressed in million kilocalories per hour or directly in terms of rupees per hour versus output in megawatts. For every generator, $(MW)_{min}$ is the minimum loading limit below which it is uneconomical to operate the unit and $(MW)_{max}$ is the maximum output limit. The analytical operating cost can be written as $C_i(P_{Gi})$ Rs/hour at output P_{Gi}, where the suffix i stands for the unit number. It generally suffices to fit a second degree polynomial [3], i.e.,

$$C_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i \text{ Rs/hour} \qquad \text{......... (3.6)}$$

The slope of the cost curve, i.e., $\dfrac{dC_i}{dP_{Gi}}$ is called the incremental fuel cost (IC), and is expressed in units of rupees per megawatt hour (Rs/MWh). If the cost curve is approximated as in (2.8) then the expression for IC will be

$$(IC)_i = a_i P_{Gi} + b_i \qquad \text{......... (3.7)}$$

i.e. a linear relationship.

3.5.2 Loss Minimization Function

Besides, the cost minimization function defined above there can be another type of objective function, which is known as loss minimization problem. The objective here is to minimize the real or active power loss on the various branches (paths) in the system. This problem can be expressed in two different ways

3.5.2.1 By summation of branch losses of all branches,

$$F_{Loss} = \sum_{l=1}^{NB} F_{Loss_l} \qquad \text{....... (3.8)}$$

Where, NB = Number of Branches in the system(area).

$$F_{Loss_l} = P_{ij} + P_{ji}; \qquad \text{....... (3.9)}$$

branch l lies between nodes i and j.

3.5.2.2 By summation of active nodal powers over all nodes of the network.

In this type of the problem, the losses of the entire network can be computed and not for the sub network. The computation of total losses is very similar to computation of the total cost. The total network losses are given when all active nodal powers are added. Hence, the total losses can be computed as

$$F_{loss} = \sum_{i=1}^{N} P_i \qquad \text{....... (3.10)}$$

Where N = Number of network nodes. It is important to note that the **slack is always included** in the total loss objective function.

3.6 OPTIMAL LOAD FLOW SOLUTION

The solution technique is based on load flow solution by NR method, Ref [6], a first order gradient adjustment algorithm for minimizing the objective function and use of penalty functions to account for inequality constraints on dependent variables.

3.6.1 Newton OPF Without Inequality Constraints

The objective function to be minimized is the operating cost

$$C = \sum_i C_i(P_{Gi}) \qquad \text{........... (3.11)}$$

subject to the load flow equations

$$\left.\begin{aligned} P_i - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos\left(\theta_{ij} + \delta_i - \delta_j\right) = 0 \\ Q_i + \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(\theta_{ij} + \delta_i - \delta_j\right) = 0 \end{aligned}\right\} \text{For each PQ bus}$$

and

$$P_i - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos\left(\theta_{ij} + \delta_i - \delta_j\right) = 0 \text{ for each PV bus.} \qquad \text{... (3.12)}$$

It is to be noted that at the i^{th} bus

$$\begin{aligned} P_i &= P_{Gi} - P_{Di} \\ Q_i &= Q_{Gi} - Q_{Di} \end{aligned} \qquad \text{...... (3.13)}$$

Where P_{Di} and Q_{Di} are load demands at bus i.

Equations (3.12) can be expressed in vector form

$$
\mathbf{f}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \left. \begin{matrix} P_i - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} + \delta_i - \delta_j) = 0 \\ Q_i + \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} + \delta_i - \delta_j) = 0 \end{matrix} \right\} & \text{For each PQ bus} \\ & \text{For each PV bus} \\ P_i - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} + \delta_i - \delta_j) = 0 & \end{bmatrix}
$$

...... (3.14)

Where the vector of dependent variables is

$$
\mathbf{x} = \begin{bmatrix} \left. \begin{matrix} |V_i| \\ \delta_i \end{matrix} \right\} & \text{For each PQ bus} \\ \delta_j & \text{For each PV bus} \end{bmatrix}
$$

.... (3.15)

and the vector of independent variables is

$$
\mathbf{y} = \begin{bmatrix} \left. \begin{matrix} |V_1| \\ \delta_1 \end{matrix} \right\} & \text{For slack bus} \\ \left. \begin{matrix} P_i \\ Q_i \end{matrix} \right\} & \text{For each PQ bus} \\ \left. \begin{matrix} P_i \\ |V_i| \end{matrix} \right\} & \text{For each PV bus} \end{bmatrix}
$$

$$
\mathbf{y} = \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} \qquad \text{...... (3.16)}
$$

In the above formulation, the objective function should include the slack bus power.

The vector of independent variables **y** can be partitioned into two parts – a vector **u** of control variables, which are to be varied to achieve optimum value of the objective function and a vector **p** of fixed, or disturbance or uncontrollable parameters. Control parameters may be voltage magnitudes on PV buses, P_{Gi} at buses with controllable power etc.

3.6.2 Problem Formulation

The optimization problem can now be restated as

$$\min \quad C(\mathbf{x},\mathbf{u}) \qquad \text{...... (3.17)}$$

subject to equality constraints

$$\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p}) = 0 \qquad \text{..... (3.18)}$$

To solve the optimization problem, defining the Lagrange function as

$$£(\mathbf{x},\mathbf{u},\mathbf{p}) = C(\mathbf{x},\mathbf{u}) + \boldsymbol{\lambda}^{T}\,\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p}) \qquad \text{..... (3.19)}$$

Where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers of the same dimension as $\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p})$.

The necessary conditions to minimize the unconstrained Lagrange function are

$$\frac{\partial £}{\partial \mathbf{x}} = \frac{\partial C}{\partial \mathbf{x}} + \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]^{T} \boldsymbol{\lambda} = 0 \qquad \text{..........(3.20)}$$

$$\frac{\partial £}{\partial \mathbf{u}} = \frac{\partial C}{\partial \mathbf{u}} + \left[\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right]^{T} \boldsymbol{\lambda} = 0 \qquad \text{..........(3.21)}$$

$$\frac{\partial £}{\partial \boldsymbol{\lambda}} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p}) = 0 \qquad \text{.......(3.22)}$$

Where **f**, **x**, **u**, **p** and $\boldsymbol{\lambda}$ are vectors.

Then (3.22) is obviously the same as equality constraints. The expressions for $\frac{\partial C}{\partial \mathbf{x}}$ and $\frac{\partial \mathbf{f}}{\partial \mathbf{u}}$ as needed in (3.20) and (3.21) are rather involved. $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ is nothing but Jacobian matrix.

Equations (3.11), (3.20) and (3.22) are non-linear algebraic equations and can only be solved iteratively. A simple yet efficient iteration scheme, that can be employed, is the steepest descent method (also called gradient method). The basic technique is to adjust the control vector **u**, so as to move from one feasible solution point in the direction of steepest descent (negative gradient) to a new feasible solution point with a lower value of objective function. By repeating these moves in the direction of the negative gradient, the minimum will finally be reached.

3.7 INEQUALITY CONSTRAINTS ON CONTROL VARIABLES AND POWER QUALITY

The control variables are always constrained. Constraints on the control variables are helpful in determining the quality of power supplied. If the tolerances on these variables are strict, the better will be the quality of power;

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}$$

e.g.

$$P_{Gi,\min} \leq P_{Gi} \leq P_{Gi,\max} \qquad \ldots (3.23)$$

If the correction Δu_i causes u_i to exceed one of the limits, u_i is set equal to the corresponding limit, i.e.,

$$\mathrm{u}_{i,\text{new}} = \begin{cases} u_{i,\max} & \textit{if} \quad u_{i,old} + \Delta u_i > u_{i,\max} \\ u_{i,\min} & \textit{if} \quad u_{i,old} + \Delta u_i < u_{i,\min} \\ u_{i,old} + \Delta u_i & \textit{otherwise} \end{cases} \qquad \ldots\ldots\ldots (3.24)$$

After a control variable reaches any of the limits, its component in the gradient should continue to be computed in later iterations, as the variable may come within limits at some later stage.

The necessary conditions for minimization of £ under constraint (3.23) are:

$$\frac{\partial £}{\partial u_i} = 0 \quad if \quad u_{i,\min} < u_i < u_{i,\max}$$
$$\frac{\partial £}{\partial u_i} \le 0 \quad if \quad u_i = u_{i,\max} \qquad \text{.......} (3.25)$$
$$\frac{\partial £}{\partial u_i} \ge 0 \quad if \quad u_i = u_{i,\min}$$

Therefore, the gradient vector has to satisfy the optimality condition in (3.25).

3.8 CLASSIFICATION OF OPF ALGORITHMS

The classification of OPF algorithms into classes is governed by the fact that powerful methods exist for the solution of the load flow. These provide easy access to intermediate solution through an iterative process. Further, it is pertinent to note that the optimum solution is close to existing load flow solution.

Therefore, one class of algorithm relies on the solved load flow and the tools provided by the load flow. The second class on the other hand has a rigorous formulation; it employs the exact optimality conditions and uses techniques to fulfill this. Here it is important to note that load flow solution is not a prerequisite. Hence, OPF algorithms can be described as:

3.8.1 **Class A:** In this method optimization starts from a solved solution of load flow. The Jacobian and other sensitivity relations are used in the optimization procedure. The entire process is iterative and after each iteration the load flow is solved afresh.

3.8.2 **Class B:** Here the method relies on the exact optimality conditions, wherein the load flow relations are attached as equality constraints. There is no prior knowledge of load flow solution. The process is iterative and each intermediate solution approaches the load flow solution.

CHAPTER 4

TEST RESULTS AND ANALYSIS

4.1 INTRODUCTION

With the growing demand for power coupled with the economic development in our country and the failing health of the SEB's the government has embarked on the process of de-regulating the power sector. This as a policy will be affecting the transmission services sector too, Ref [9]. Already in some of the states, SEB's have undergone restructuring, which has resulted into formation of corporations dealing with generation, transmission and distribution of power. Due to this change, there arose a need for having a mechanism, which could provide remunerative prices to the industry and at the same time, be able to attract fresh investments for strengthening this core sector.

Moreover, with the movement of the industry from a single part tariff to multi part tariff covering the fixed and the variable costs, this method of arriving at the variable cost component by way of nodal price calculation is a step toward fair charging to utilities based on their usage of the assests.

It is in this perspective that, this work has been carried out, the emphasis being, to put forward a way for pricing of the power in the transmission sector. In this work, the price of wheeling power over a network has been calculated with, different types of objective functions and then arriving at an optimum cost in each of the case.

While arriving at the cost of wheeling of power on the transmission network, following variations have been considered:

a) Pricing of the real (active) power only:

   a. With standard taps settings (including the cost of tap changing),

   b. With optimized tap settings (including the cost of tap changing),

b) Pricing of the real power and reactive power with optimized tap settings (including the cost of tap changing):

   i. Reactive power price as represented by Sinh curve,

   ii. Reactive power price as represented by Inverse Tanh curve,

c) Pricing of the real power, reactive power, with optimal placement of capacitor banks at the load buses (including the cost of tap changing):

   i. Reactive power price as represented by Sinh curve,

   ii. Reactive power price as represented by Inverse Tanh curve.

Besides, another study was carried out by changing the real power loading on the buses in case of 14 Bus system. The load was changed in the steps of 4% to find out the behavior of the system as regards to the effect on the bus marginal prices.

4.2 MODELING FOR COST OF REAL POWER

The real power price has been taken as a quadratic function which can be depicted as

$$C_{Pgi} = a_i + b_i Pg_i + c_i Pg_i^{\,2}, \text{ Rs/hour} \;\;\ldots\ldots(4.1)$$

Where a, b and c are the coefficients of the cost curve for a given generator i, and C_{Pgi} represents the cost of the producing power at the generator i. The cost curve for a given generator is derived from the incremental heat rate curve for that generator.

4.3 MODELING FOR COST OF REACTIVE POWER

Considering the possibility of a generator going in for production of leading Vars for some time, or a generator biding for reactive power supply, the cost curve for the generator / synchronous capacitor was modeled as a polynomial function based on the Taylor's series expression of sinh and inverse tan hyperbolic functions. These were used as they are continuous, as also they depict the nature of the price curve. Moreover, the same curve can also be used for the

pricing of the both leading and lagging reactive power production for the generator.

The Taylor series expansion for Sinh and Tanh^{-1} are as given below:

$$\text{Sinh}\, x = x + \frac{x^3}{6} + \frac{x^5}{120} \qquad \ldots\ldots (4.2)$$

$$\text{Tanh}^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} \qquad \ldots\ldots (4.3)$$

4.4 MODELING THE COST OF TAP CHANGES ON THE TAP CHANGING UNDER LOAD TRANSFORMER (TCUL)

The model takes into account the operation of the transformers fitted with tap changer. The tap changer is assumed to be working under load, i.e. the transformers are of type TCUL. In addition, taps settings are modified to arrive at an optimum solution and to arrive at an optimum price of transmission.

Say for a, 132/33 KV, 100 MVA transformer costs Rs.400,00,000/- and its taps can be adjusted 157388 times during its lifetime (= 11 steps/day * 365 days/yr * 40 years * 0.98 availability factor). Therefore, cost of each step change is Rs.254.15. If each step change corresponds to 5/8% tap ratio change then, the cost of 1% tap ratio change is Rs. 406.60 (= 254.15/0.625).

4.5 MODELING FOR OPTIMUM PLACEMENT OF CAPACITOR BANKS AND THEIR COSTING

We are well aware that at the load buses due to the fluctuation of the load, the voltage is adversely affected. Mostly, Voltage is depressed at the load end in our country due to more than the desired load connected at the consumer end. This problem can be obviated by the placement of capacitor banks or any other device like a TSC or SVC. These devices help in propping up the voltage and reducing the loss.

The program using the optimization routine finds out the capacitor placement at the load buses. Once the optimum capacitor placement is found, the

cost is attributed for the use of capacitor. Since, these equipments, like any other equipment have a given life in terms of the number of cycles of operation / time. Therefore, as the number of operations done on them increases, it leads to wear and tear and thus affecting their life adversely. E.g., repeated operations on the capacitor banks, TSC or SVC will result in the deterioration of the dielectric medium and the operating gear. This may lead to shorter life if these operations are resorted to more frequently.

Taking this approach, the costing for the component of price attributed to the capacitors is as discussed below:

Say a 66 KV, 1 MVAR capacitor costs Rs. 5,00,000. It can be switched 30,047 times (=12 switching operations/day * 365 days/yr * 7 yrs * 0.98 availability factor). So the depreciated cost is Rs.16.64/switching operation/ MVAR (Rs.500, 000/30047). If the capacity of the capacitor is 5 MVAR then the cost will be Rs. 83.2 (=5 * 16.64).

4.6 PROBLEM FORMULATION

In view of the presence of above elements in the transmission system, an exercise to evaluate, the minimum optimal cost in an event of dispatch of power over the network was carried out. The aim was to find out the cost of transaction of power between two buses or nodes. This simulation was carried out on IEEE standard 14 Bus system taking into account the various objective functions. The problem can be defined as:

Minimize,

$$\begin{aligned} C_{Txn} = & \sum_{i=1}^{NG} \left(a + bPgi + cPgi^2\right) + \\ & \sum_{i=1}^{NT} \left(Tapsetting \times Tap\ Cost\right) + \\ & \sum_{i=1}^{NB} \left(Qc_i \times Cap\ Cost\right) \end{aligned} \qquad \ldots\ldots (4.4)$$

s.t.

a) $$Pi = Pgi - Pdi = |Vi| \sum_{j=1}^{NB} |Vj| \left[Gij \cos(\delta i - \delta j) + Bij \sin(\delta i - \delta j)\right], \ldots\ldots (4.5)$$

1. for $i = 1 \ldots NB$,

b) $$Qi = Qgi - Qdi = |Vi| \sum_{j=1}^{NB} |Vj| [Gij \sin(\delta i - \delta j) - Bij \cos(\delta i - \delta j)], \ldots\ldots (4.6)$$

1. for $i = 1, \ldots NG$,

c) $$Qi = Qgi - Qdi + Qci = |Vi| \sum_{j=1}^{NB} |Vj| [Gij \sin(\delta i - \delta j) - Bij \cos(\delta i - \delta j)]$$

for $i = 1, \ldots NL$ …… (4.7)

Where

C_{Txn} = Cost of Transmitting power,

NG = Number of generators,

NT = Number of tap transformers,

NB = Total number of buses;

NL = Number of Load buses;

Qci = Reactive support provided by way of capacitors;

Besides the above-mentioned constraints, in real power systems variables have binding limits. These are required least they may damage the equipment or bring the system into unstable, insecure operating state. Hence, the systems have additional constraints such as:

b. Limit on active (real) power of a generator (PV Node):

$$P_{gi_{low}} \le P_{gi} \le P_{gi_{max}} \qquad \ldots\ldots (4.8)$$

c. Limit on reactive power generation of a PV node:

$$Q_{gi_{low}} \le Q_{gi} \le Q_{gi_{max}}, \qquad \ldots\ldots (4.9)$$

in reality the reactive power limits of a generator are complex and state dependent. This is a simplification of the limits.

d. Limit on Voltage of a PV and PQ (load bus) Node:

$$|V_i| = Constant \text{ for } i = 1, \ldots NG \qquad \ldots\ldots (4.10)$$

$$|V|_{Low_i} \le |V|_i \le |V|_{High_i} \text{ for } i = 1, \ldots NL, \qquad \ldots\ldots (4.11)$$

e. Limit on tap positions of a transformer:

$$t_{low_i} \le t_i \le t_{high_i} \qquad \ldots\ldots (4.12)$$

f. Limit on capacitance at a PQ (load bus) node:

$$Q_{ci_{low}} \le Q_{ci} \le Q_{ci_{max}} \qquad \ldots\ldots (4.13)$$

g. Limits on MVA flow in the transmission lines or transformers:

$$P_{ij}^2 + Q_{ij}^2 \le S_{high_{ij}}^2 \qquad \ldots\ldots (4.14)$$

h. Upper limit on the active power flow in transmission lines or transformers: $P_{ij} \le P_{high_{ij}}$ ...... (4.15)

Besides these there can be still more constraints that can be used effectively but have not been considered in this study.

4.7 SOLUTION OF THE OPTIMIZATION PROBLEM

GAMS was used to arrive at the optimum cost for transmission of power for the cases. The optimization routine and the code was written in GAMS. The problem was approached by NLP (non-linear programming) model using CONOPT / CONOPT2 solver. The code was executed on the windows environment with PIII with a speed of 667 Mhz.

4.8 TEST RESULTS – IEEE STANDARD 14 BUS SYSTEM

4.8.1 Case 1 - Pricing of Real Power only.

4.8.1.1 Standard tap setting. To begin with a situation was assumed were, the cost of wheeling of real power is only charged by the transmission company (Transco). The associated reactive power at the given pf is allowed to move free of cost. The system operates at the given tap settings. It was found that the voltage ranges from 1.029 to 1.056 at the load buses. The cost of moving power from one bus to the other is the difference of the marginal costs at these buses (nodes). The system operation cost works out to 156.3695. The details of the result are attached as appendix A, tables A.1 to A.4.

4.8.1.2 **<u>Optimized tap setting.</u>** In the above system, the taps too were considered as variable and the program was allowed to determine the optimum value of the taps such that the operating costs were minimum. Hence, the system adjusted the values of the taps as 0.9, 0.9 and 0.9 respectively for all the taps. This reduced the cost to 156.2018 but in doing so, the reactive power losses did increase. The results are attached as appendix A, tables A.5 to A.8.

4.8.2 <u>Case 2 – Pricing of Real and Reactive power with optimized taps and reactive power price as a function of sinh x.</u>

4.8.2.1 In this case, the system's objective function was changed. The price of the reactive power and optimization of taps was also now considered, besides the price of real power. This led to a marginal increase in bus marginal price as compared to case 1 and also in the overall cost function, which returned a value of 156.5787. This increase in the value is due to the reactive power price curve being modeled as a function of sinh x. The details of the result is attached as appendix A, tables A.9 to A.12.

4.8.3 <u>Case 3 – Pricing of Real and Reactive power with optimized taps and reactive power price as a function of inverse tan hyperbolic x.</u>

4.8.3.1 Here the system was studied with the same parameters as in the case 2 above, but the reactive power price function was modeled as a function of inverse tan hyperbolic x. It was noticed that there was a subtle change in the values of bus marginal prices, besides an increase in the overall cost function. The detailed results are attached as appendix A, tables A.13 to A.16.

4.8.4 <u>Case 4 – Pricing of Real and Reactive power with optimized tap setting and addition reactive support at load bus price.</u>

4.8.4.1 In this case, the system was allowed to provide reactive support at the load buses. It was seen that, with addition of capacitors at the load buses the voltage profile at the buses improved and the losses both real and reactive went down. There was an increase in the overall cost through a small value but there was a reduction in the bus marginal price, as compared with bus marginal prices as obtained in case 2 above. Thus, the cost of moving power would be less in this case between two buses. The reactive power was modeled as a function of sinh x. Detailed results is attached in appendix A, tables A.17 to A.21.

4.8.4.2 In the second case, the only change made was that, now the reactive power price was now modeled as inverse tan hyperbolic function. Here too the results were as discussed in Para (d) (i) above. Besides, this system IEEE 30 Bus System was also studied along the same lines and the results for the same have been reflected in appendix A, tables A.22 to A.26.

4.9 <u>TEST RESULTS – IEEE STANDARD 14 BUS SYSTEM WITH LOAD CHANGE</u>

In this study, the system mentioned above was put through load change by adding a load-scaling factor (LSF). The real power load was varied in the system. The objective function being, minimization of cost. Price for both real and reactive power was taken into account and the reactive power cost was modeled as a function of inverse tan hyperbolic. Besides this, the system was allowed to add reactive support as obtained by the optimization routine. To simulate the conditions as in actual the additional reactive support was limited to a maximum of 0.5 pu MVAR on a base of 100 MVA at any bus.

Load was varied in steps of 4% each. It was found that he the system could supply an additional load of up to 16%, (refer appendix A1, tables A1.1 to A1.16) with addition of a maximum of 0.042 pu MVAR of reactive support at bus number 13.

During this exercise ceiling limits on real and reactive power of generators, were not altered. The system could not supply power any further on increase of load, due to the shortage of real power.

Another important result was that, when the reactive support as obtained in case 4 (ii) were kept fixed (i.e. capacitors support at load bus number 13 and 14 = 0.031 pu MVAR) then, the load on the system could supply an increased load of up to 15%. (Refer Appendix A2).

4.10 TEST RESULTS - IEEE STANDARD 30 BUS SYSTEM

As in the case of IEEE Standard 14 Bus System, results were compiled for 30-bus system too. The deductions are similar to those in the case of 14 bus. The results have been attached as appendix B, tables B.1 to B.19.

4.11 ANALYSIS

On studying the abovementioned system in respect of minimizing the cost of generation, it was seen that:

4.11.1 The cost arrived at by the Newton Raphson method varied at each node (bus) in the system. The marginal costs were high for the buses, which were distant from the generators.

4.11.2 The cost function as modeled on sinh x function returned a lower value of cost of generation as also the price of transmission between two buses, in comparison to the cost function modeled by inverse tanh function.

4.11.3 The cost of generation and hence the price of transmission between two nodes decreased with the use of capacitors (refer table). The determination for the placement of the capacitors, including their value was done using optimization schedule.

4.11.4 With the placement of capacitors, there was a reduction in the system losses. Further, with these capacitors in place the same system could supply an additional load of up to 15% than in the case with no capacitors.

4.11.5 There was a lower cost of transmission in the system with adequate capacitors, in comparison to the same system with no capacitors.

CHAPTER 5

CONCLUSIONS AND SCOPE FOR FURTHER WORK

5.1 CONCLUSIONS

The study was carried out on IEEE standard 14 and 30 bus systems. The aim of the study was to determine the impact on cost of transmission with reference to different functions representing the pricing of reactive power. Further, a comparison was made in the same study to see the effect of equipments like capacitors and the tap changers on the cost of transmission.

It needs to be highlighted here that when the cost for transmission was found without the addition of reactive support it worked out to be more than with the addition of capacitors. Also, with the addition of capacitors another benefit accrued, the same line could be now supply more load than in the earlier case i.e. without reactive support.

Moreover, the increased capital cost in terms of the installation of these capacitors can be recovered by way of less line losses occurring on the lines which are compensated by the capacitors. The lines needing compensation and also the extent to which it was required was calculated by the use of Optimal Power Flow routine, which also calculated the positions of the taps on a tap changer. Now

with the movement of the utilities from single part tariff to tariffs based on fixed and variable component, this system offer a means to calculating price for transmission based on the drawls of energy by a utility from the transmission system.

In arriving at the solution, for determining the cost of transmission, the objective function has been taken as minimizing the cost of generation. The OPF solution is based on Class B type of algorithm of OPF.

5.2 SCOPE FOR FURTHER WORK

- The work can be extended to include the bidding process by the generators for the supply of power in terms of various operating conditions like backing down, provision of reactive power.
- Further work can take into account the switching of the capacitor banks in integer values, rather than as continuous devices.
-

DATA FOR 14 BUS SYSTEM

The IEEE 14 bus system is shown in Fig A.1.

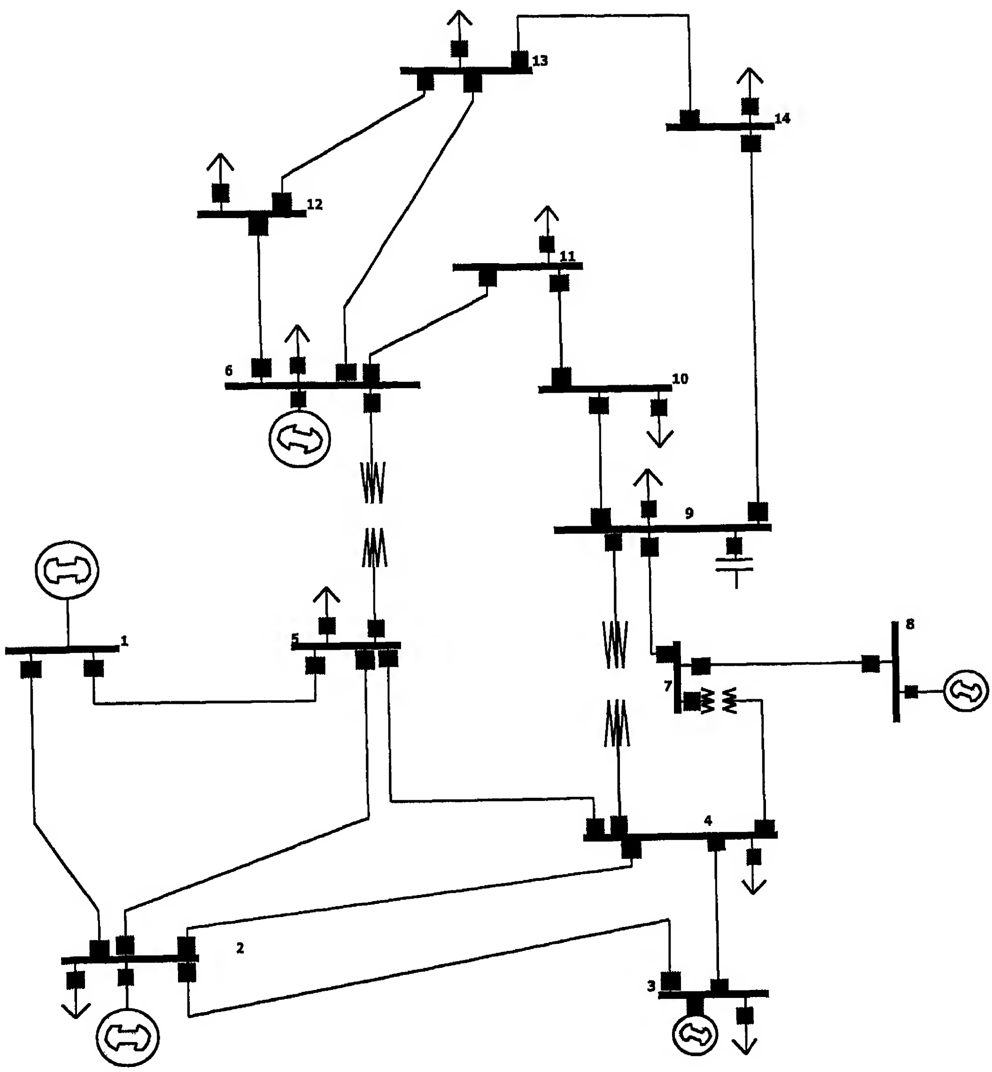


Fig A.1 14 bus system

The relevant data are provided in the following tables.

1. Table A.1 gives Generator Bus Voltages.
2. Table A.2 gives Generator Data.
3. Table A.3 gives Transformer Data.
4. Table A.4 gives Load Bus data.
5. Table A.5 gives Line Data.
6. Table A.6 gives Generator Cost Characteristics

Note:

1. Base MVA = 100 MVA

Table A.1: Generator Bus Voltages for 14 bus system

Generator No	Real Power Generation Limit		Reactive Power Generation Limit	
	Maximum (MW)	Minimum (MW)	Maximum (MW)	Minimum (MW)
1	332.40	0.0	100.0	-45.0
2	140.0	0.0	050..0	-40.0
3	100.0	0.0	040.0	00.0
6	100.0	0.0	060.4	-06.0
8	100.0	0.0	024.0	-06.0

Table A.2: Generator Data for 14 bus system

Bus No.	Scheduled Real Power Generation P_G(MW)	Specifieed Voltage Magnitude V_{spec}(p.u)	Load	
			Real (MW)	Reactive (MVAR)
1	-	1.060	00.00	00.00
2	40.0	1.045	21.70	12.70
3	-	1.010	94.20	19.00
6	20.0	1.070	11.2	7.5
8	-	1.090	00.00	00.00

Table A.3: Transformer Data for 14 bus system

Line No	From Bus	To Bus	Series Impedence		Tap Setting
			Resistance (0.u)	Reactance (p.u)	
8	4	7	0.0	0.2091	0.978
9	4	9	0.0	0.5561	0.969
10	5	6	0.0	0.2502	0.962

Table A.4 Load Bus Data for 14 bus system

Bus no	Load		External Shunt Susceptance (p.u)
	Real	Reactive	
4	47.8	4.0	0.0
5	7.6	1.6	0.0
7	0.0	0.0	0.0
9	29.5	16.6	0.19
10	9.0	5.8	0.0
11	3.5	1.8	0.0
12	6.1	1.6	0.0
13	13.5	5.8	0.0
14	14.9	5.0	0.0

Table A.5: Line Data for 14 bus system

Line. No.	From No	To Bus	Series Impedance		Shunt Susceptance (p.u)
			Resistance (p.u)	Reactance (p.u)	
1	1	2	0.01938	0.05917	0.528
2	1	5	0.05403	0.22304	0.0492
3	2	3	0.04699	0.19797	0.0438
4	4	4	0.05811	0.17632	0.0374
5	2	5	0.05695	0.17388	0.0340
6	3	4	0.06701	0.17103	0.0346
7	4	5	0.01335	0.04211	0.0
11	6	11	0.09798	0.19890	0.0
12	6	12	0.12291	0.25581	0.0
13	6	13	0.06615	0.13027	0.0
14	7	8	0.0	0.17615	0.0
15	7	9	0.0	0.11001	0.0
16	9	10	0.03181	0.08450	0.0
17	9	14	0.12711	0.27038	0.0
18	10	11	0.08205	0.19207	0.0
19	12	13	0.22092	0.19988	0.0
20	13	14	0.17093	0.34802	0.0

Table A. 6: Generator Cost Characteristics

Generator at Bus Number	Generator cost Characteristics			Remarks
	a	b	c	
1	0	50	0.0005	
2	0	100	0.001	
3	0	150	0.0015	
6	0	170	0.0017	
8	0	40	0.0004	

RESULTS FOR 14 BUS SYSTEM

Case 1: Bus operating conditions with cost minimization

Table A.1: Real Power Cost Minimization with specified tap settings

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost	Remarks
1	1.060	0.000	50.002	
2	1.045	-4.950	52.729	
3	1.010	-12.611	56.759	
4	1.029	-10.416	55.522	
5	1.035	-8.952	54.659	
6	1.070	-14.666	54.732	
7	1.056	-13.548	55.472	
8	1.090	-13.548	55.472	
9	1.050	-15.163	55.456	
10	1.046	-15.360	55.640	
11	1.054	-15.139	55.364	
12	1.055	-15.510	55.611	
13	1.049	-15.569	55.892	
14	1.032	-16.347	56.803	

Objective function:

Minimize Cost of Real Power

Value of objective function = 156.3695

Total Line Losses:

Real power loss = 0.1337531 pu
Reactive power loss = 0.551626 pu

Table A.2: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)	
8	4	7	0.9	1.1	0.978
9	4	9	0.9	1.1	0.969
10	5	6	0.9	1.1	0.962

Table A.3: Generation Detail.

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.324	-0.235	
2	0.400	0.274	
3		0.180	
6		0.404	
8		0.211	

Table A.4: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.95	20.19	151.71	-33.07	157.25	200
1	5	-76.42	3.34	73.62	-14.81	76.50	200
2	3	-72.53	-3.63	70.25	-5.81	72.62	200
2	4	-56.01	8.34	54.32	-13.42	56.62	200
2	5	-41.47	7.87	40.55	-10.63	42.21	200
3	4	23.95	2.33	-24.33	-3.32	24.06	200
4	5	62.31	-7.39	-62.81	5.83	62.75	200
4	7	-29.05	0.83	29.05	-2.42	29.06	200
4	9	-16.60	-3.12	16.60	1.72	16.89	200
5	6	-46.95	-20.14	46.95	14.80	51.09	200
6	11	-7.11	-5.07	7.05	4.94	8.73	200
6	12	-7.80	-2.70	7.73	2.55	8.25	200
6	13	-17.65	-7.99	17.44	7.57	19.38	200
7	8	0.00	20.45	0.00	-21.12	20.45	200
7	9	-28.41	-6.35	28.41	5.52	29.11	200
9	10	-5.48	-2.73	5.47	2.70	6.12	200
9	14	-9.52	-2.65	9.41	2.41	9.88	200
10	11	3.53	3.10	-3.55	-3.14	4.70	200
12	13	-1.63	-0.95	1.62	0.95	1.88	200
13	14	-5.55	-2.71	5.49	2.59	6.18	200

Case 1b: Bus operating conditions with cost minimization

Table A.5: Real Power Cost minimization with optimized tap settings

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost	Remarks
1	1.060	0.000	50.002	
2	1.045	-4.950	52.729	
3	1.010	-12.611	56.759	
4	1.029	-10.416	55.522	
5	1.035	-8.952	54.659	
6	1.070	-14.666	54.732	
7	1.056	-13.548	55.472	
8	1.090	-13.548	55.472	
9	1.050	-15.163	55.456	
10	1.046	-15.360	55.640	
11	1.054	-15.139	55.364	
12	1.055	-15.510	55.611	
13	1.049	-15.569	55.892	
14	1.032	-16.347	56.803	

Objective function:

Minimize Cost of Real Power

Value of objective function = 156.2018

Total Line Losses:

Real power loss = 0.1337532 pu

Reactive power loss = 0.618426 pu

Table A.6: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.911

Table A.7: Generation Detail.

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.324	-0.235	
2	0.400	0.274	
3		0.180	
6		0.404	
8		0.211	

Table A.8: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.95	20.19	151.71	-33.07	157.25	200
1	5	-76.42	3.34	73.62	-14.81	76.50	200
2	3	-72.53	-3.63	70.25	-5.81	72.62	200
2	4	-56.01	8.34	54.32	-13.42	56.62	200
2	5	-41.47	7.87	40.55	-10.63	42.21	200
3	4	23.95	2.33	-24.33	-3.32	24.06	200
4	5	62.31	-7.39	-62.81	5.83	62.75	200
4	7	-29.05	0.83	29.05	-2.42	29.06	200
4	9	-16.60	-3.12	16.60	1.72	16.89	200
5	6	-46.95	-20.14	46.95	14.80	51.09	200
6	11	-7.11	-5.07	7.05	4.94	8.73	200
6	12	-7.80	-2.70	7.73	2.55	8.25	200
6	13	-17.65	-7.99	17.44	7.57	19.38	200
7	8	0.00	20.45	0.00	-21.12	20.45	200
7	9	-28.41	-6.35	28.41	5.52	29.11	200
9	10	-5.48	-2.73	5.47	2.70	6.12	200
9	14	-9.52	-2.65	9.41	2.41	9.88	200
10	11	3.53	3.10	-3.55	-3.14	4.70	200
12	13	-1.63	-0.95	1.62	0.95	1.88	200
13	14	-5.55	-2.71	5.49	2.59	6.18	200

Case 2: Real and Reactive Power cost minimization and optimized taps

Table A.9: Reactive Cost Function modeled as Sinh x

Bus Number	Voltage (p.u.)	Angle (Degree)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.002	-0.479	
2	1.045	-4.950	53.057	0.562	
3	1.010	-12.611	57.164	0.364	
4	1.029	-10.416	55.893	0.284	
5	1.035	-8.952	54.983	0.324	
6	1.070	-14.666	55.212	0.852	
7	1.056	-13.548	55.880	0.555	
8	1.090	-13.548	55.880	0.429	
9	1.050	-15.163	55.887	0.778	
10	1.046	-15.360	56.089	1.002	
11	1.054	-15.139	55.833	1.031	
12	1.055	-15.510	56.125	1.123	
13	1.049	-15.569	56.402	1.301	
14	1.032	-16.347	57.308	1.436	

Objective function:

Minimize Cost of Real Power

Value of objective function = 156.5787

Total Line Losses:

Real power loss = 0.1337538 pu
Reactive power loss = 0.618426 pu

Table A.10: Optimized Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.911

Table A.11: Generation Detail.

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.324	-0.235	
2	0.400	0.274	
3		0.180	
6		0.404	
8		0.211	

Table A.12: Line Flows (Sinh x)

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.95	20.19	151.71	-33.07	157.25	200
1	5	-76.42	3.34	73.62	-14.81	76.50	200
2	3	-72.53	-3.63	70.25	-5.81	72.62	200
2	4	-56.01	8.34	54.32	-13.42	56.62	200
2	5	-41.47	7.87	40.55	-10.63	42.21	200
3	4	23.95	2.33	-24.33	-3.32	24.06	200
4	5	62.31	-7.39	-62.81	5.83	62.75	200
4	7	-31.57	-49.00	31.57	43.57	58.29	200
4	9	-17.87	-20.12	17.87	17.04	26.91	200
5	6	-48.02	-32.00	48.02	25.49	57.71	200
6	11	-7.11	-5.07	7.05	4.94	8.73	200
6	12	-7.80	-2.70	7.73	2.55	8.25	200
6	13	-17.65	-7.99	17.44	7.57	19.38	200
7	8	0.00	20.45	0.00	-21.12	20.45	200
7	9	-28.41	-6.35	28.41	5.52	29.11	200
9	10	-5.48	-2.73	5.47	2.70	6.12	200
9	14	-9.52	-2.65	9.41	2.41	9.88	200
10	11	3.53	3.10	-3.55	-3.14	4.70	200
12	13	-1.63	-0.95	1.62	0.95	1.88	200
13	14	-5.55	-2.71	5.49	2.59	6.18	200

Case 2b: Real Power and Reactive Power cost minimization with optimized tap settings

Table A.13: Reactive Power cost modeled as Inverse Tanh x

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.002	-0.489	
2	1.045	-4.950	53.065	0.578	
3	1.010	-12.611	57.172	0.368	
4	1.029	-10.416	55.904	0.300	
5	1.035	-8.952	54.994	0.342	
6	1.070	-14.666	55.233	0.908	
7	1.056	-13.548	55.892	0.575	
8	1.090	-13.548	55.892	0.435	
9	1.050	-15.163	55.900	0.806	
10	1.046	-15.360	56.104	1.036	
11	1.054	-15.139	55.851	1.076	
12	1.055	-15.510	56.147	1.178	
13	1.049	-15.569	56.424	1.354	
14	1.032	-16.347	57.327	1.476	

Objective function:

Minimize Cost of Real Power

Value of objective function = 156.5860

Total Line Losses:

Real power loss = 0.1337538 pu
Reactive power loss = 0.618426 pu

Table A.14: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.911

Table A.15: Generation Detail.

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.324	-0.235	
2	0.400	0.274	
3		0.180	
6		0.404	
8		0.211	

Table A.16: Line Flows (inv. Tanh x)

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.95	20.19	151.71	-33.07	157.25	200
1	5	-76.42	3.34	73.62	-14.81	76.50	200
2	3	-72.53	-3.63	70.25	-5.81	72.62	200
2	4	-56.01	8.34	54.32	-13.42	56.62	200
2	5	-41.47	7.87	40.55	-10.63	42.21	200
3	4	23.95	2.33	-24.33	-3.32	24.06	200
4	5	62.31	-7.39	-62.81	5.83	62.75	200
4	7	-31.57	-49.00	31.57	43.57	58.29	200
4	9	-17.87	-20.12	17.87	17.04	26.91	200
5	6	-48.02	-32.00	48.02	25.49	57.71	200
6	11	-7.11	-5.07	7.05	4.94	8.73	200
6	12	-7.80	-2.70	7.73	2.55	8.25	200
6	13	-17.65	-7.99	17.44	7.57	19.38	200
7	8	0.00	20.45	0.00	-21.12	20.45	200
7	9	-28.41	-6.35	28.41	5.52	29.11	200
9	10	-5.48	-2.73	5.47	2.70	6.12	200
9	14	-9.52	-2.65	9.41	2.41	9.88	200
10	11	3.53	3.10	-3.55	-3.14	4.70	200
12	13	-1.63	-0.95	1.62	0.95	1.88	200
13	14	-5.55	-2.71	5.49	2.59	6.18	200

Case 3: Real Power and Reactive Power cost minimization with optimized tap settings and addition of reactive support at load bus

Table A.17: Reactive Power cost modeled as Sinh x

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.002	-0.482	
2	1.045	-4.949	53.053	0.554	
3	1.010	-12.608	57.157	0.360	
4	1.030	-10.421	55.878	0.242	
5	1.035	-8.952	54.967	0.286	
6	1.070	-14.638	55.174	0.751	
7	1.057	-13.558	55.863	0.492	
8	1.090	-13.558	55.863	0.413	
9	1.052	-15.172	55.867	0.672	
10	1.048	-15.363	56.064	0.892	
11	1.055	-15.128	55.802	0.924	
12	1.057	-15.509	56.079	0.963	
13	1.053	-15.639	56.355	1.000	
14	1.039	-16.486	57.249	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 156.8399

Total Line Losses:

Real power loss = 0.1332406 pu
Reactive power loss = 0.6185417 pu

Table A.18: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.983
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	1.100

Table A.19: Placement of Capacitors at load buses

Bus Number	Capacitance (pu MVAR)
13	0.026
14	0.030

Table A.20: Generation Detail

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.324	-0.235	
2	0.400	0.274	
3		0.180	
6		0.404	
8		0.211	

Table A.21: Line Flows (Sinh x)

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.91	20.18	151.67	-33.05	157.21	200
1	5	-76.41	3.46	73.60	-14.92	76.49	200
2	3	-72.51	-3.63	70.24	-5.81	72.60	200
2	4	-56.02	8.57	54.32	-13.65	56.67	200
2	5	-41.44	8.02	40.52	-10.78	42.21	200
3	4	23.96	2.55	-24.34	-3.55	24.10	200
4	5	62.47	-7.74	-62.97	6.17	62.94	200
4	7	-29.00	3.88	29.00	-5.51	29.25	200
4	9	-17.94	-19.71	17.94	16.70	26.65	200
5	6	-39.60	46.19	39.60	-56.72	60.84	200
6	11	-7.04	-4.55	6.98	4.43	8.38	200
6	12	-7.66	-1.95	7.59	1.81	7.90	200
6	13	-17.66	-5.01	17.47	4.63	18.36	200
7	8	0.00	19.75	0.00	-20.36	19.75	200
7	9	-28.50	-5.20	28.50	4.37	28.97	200
9	10	-5.55	-3.23	5.53	3.20	6.42	200
9	14	-9.60	-0.70	9.49	0.47	9.63	200
10	11	3.47	2.60	-3.48	-2.63	4.33	200
12	13	-1.49	-0.21	1.49	0.21	1.51	200
13	14	-5.46	-1.64	5.41	1.54	5.70	200

Case 3: Real Power and Reactive Power cost minimization with optimized tap settings and addition of reactive support at load bus

Table A.22: Reactive Power cost modeled as Inverse Tanh x

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.002	-0.491	
2	1.045	-4.949	53.061	0.568	
3	1.010	-12.608	57.164	0.363	
4	1.030	-10.421	55.886	0.251	
5	1.035	-8.952	54.974	0.296	
6	1.070	-14.636	55.186	0.777	
7	1.057	-13.558	55.872	0.501	
8	1.090	-13.558	55.872	0.418	
9	1.052	-15.173	55.876	0.684	
10	1.048	-15.363	56.074	0.907	
11	1.055	-15.128	55.813	0.943	
12	1.057	-15.511	56.092	0.980	
13	1.054	-15.649	56.366	1.000	
14	1.039	-16.493	57.259	1.000	

Objective function;

Minimize Cost of Real Power

Value of objective function = 156.8450

Total Line Losses:

Real power loss = 0.1332117 pu

Reactive power loss = 0.618399 pu

Table A.23: Placement of Capacitors at load buses

Bus Number	Capacitance (pu MVAR)	
13	0.031	
14	0.031	

Table A.24: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.983
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	1.100

Table A.25: Generation Detail

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.323	-0.236	
2	0.400	0.270	
3		0.178	
6		0.355	
8		0.203	

Table A.26: Line Flows (Inverse Tanh x)

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.91	20.18	151.67	-33.05	157.21	200
1	5	-76.41	3.47	73.60	-14.93	76.49	200
2	3	-72.51	-3.63	70.24	-5.81	72.60	200
2	4	-56.02	8.58	54.32	-13.66	56.67	200
2	5	-41.44	8.02	40.52	-10.78	42.21	200
3	4	23.96	2.56	-24.34	-3.56	24.10	200
4	5	62.48	-7.76	-62.97	6.19	62.96	200
4	7	-29.00	3.88	29.00	-5.51	29.26	200
4	9	-17.94	-19.69	17.94	16.68	26.64	200
5	6	-39.59	46.18	39.59	-56.71	60.83	200
6	11	-7.03	-4.53	6.98	4.41	8.37	200
6	12	-7.65	-1.86	7.58	1.72	7.87	200
6	13	-17.67	-4.66	17.48	4.28	18.27	200
7	8	0.00	19.71	0.00	-20.32	19.71	200
7	9	-28.51	-5.14	28.51	4.31	28.97	200
9	10	-5.55	-3.26	5.54	3.23	6.44	200
9	14	-9.60	-0.60	9.50	0.37	9.62	200
10	11	3.46	2.57	-3.48	-2.61	4.31	200
12	13	-1.48	-0.12	1.48	0.12	1.48	200
13	14	-5.45	-1.65	5.40	1.55	5.70	200

Table A1.2: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.911

Table A1.3: Generation Detail

Bus Number	Real power	Reactive power
1	2.324	-0.235
2	0.400	0.274
3		0.180
6		0.404
8		0.211

Table A1.4: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-155.95	20.19	151.71	-33.07	157.25	200
1	5	-76.42	3.34	73.62	-14.81	76.50	200
2	3	-72.53	-3.63	70.25	-5.81	72.62	200
2	4	-56.01	8.34	54.32	-13.42	56.62	200
2	5	-41.47	7.87	40.55	-10.63	42.21	200
3	4	23.95	2.33	-24.33	-3.32	24.06	200
4	5	62.31	-7.39	-62.81	5.83	62.75	200
4	7	-31.57	-49.00	31.57	43.57	58.29	200
4	9	-17.87	-20.12	17.87	17.04	26.91	200
5	6	-48.02	-32.00	48.02	25.49	57.71	200
6	11	-7.11	-5.07	7.05	4.94	8.73	200
6	12	-7.80	-2.70	7.73	2.55	8.25	200
6	13	-17.65	-7.99	17.44	7.57	19.38	200
7	8	0.00	20.45	0.00	-21.12	20.45	200
7	9	-28.41	-6.35	28.41	5.52	29.11	200
9	10	-5.48	-2.73	5.47	2.70	6.12	200
9	14	-9.52	-2.65	9.41	2.41	9.88	200
10	11	3.53	3.10	-3.55	-3.14	4.70	200
12	13	-1.63	-0.95	1.62	0.95	1.88	200
13	14	-5.55	-2.71	5.49	2.59	6.18	200

Case 2: Real and Reactive power cost minimization with optimized tap settings

Table A1.5: LSF = 1.04

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0	50.002	-0.527	LSF = 1.04
2	1.045	-5.218	53.260	0.657	
3	1.010	-13.216	57.584	0.414	
4	1.028	-10.894	56.230	0.364	
5	1.034	-9.363	55.267	0.404	
6	1.070	-15.316	55.523	0.931	
7	1.055	-14.157	56.225	0.614	
8	1.090	-14.157	56.225	0.444	
9	1.049	-15.839	56.236	0.853	
10	1.045	-16.046	56.448	1.080	
11	1.054	-15.814	56.177	1.112	
12	1.054	-16.199	56.482	1.206	
13	1.049	-16.265	56.774	1.386	
14	1.030	-17.080	57.729	1.522	

Objective function:

Minimize Cost of Real Power

Value of objective function = 162.5481

Total Line Losses:

Real power loss = 0.145982 pu
Reactive power loss = 0.654496 pu

Table A1.6: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	1.034
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.900

Table A1.7: Generation Detail

Bus Number	Real power	Reactive power
1	2.440	-0.252
2	0.400	0.307
3		0.202
6		0.412
8		0.215

Table A1.8: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-164.08	22.07	159.37	-36.36	165.56	200
1	5	-79.88	3.14	76.81	-15.68	79.94	200
2	3	-75.57	-3.34	73.10	-6.92	75.64	200
2	4	-58.20	8.23	56.38	-13.71	58.78	200
2	5	-43.03	7.67	42.05	-10.63	43.71	200
3	4	24.87	1.30	-25.28	-2.36	24.90	200
4	5	64.88	-8.20	-65.42	6.50	65.40	200
4	7	-28.57	27.90	28.57	-31.34	39.93	200
4	9	-18.58	-20.06	18.58	16.88	27.35	200
5	6	-50.60	-38.52	50.60	30.80	63.59	200
6	11	-7.40	-5.17	7.34	5.03	9.03	200
6	12	-8.11	-2.72	8.03	2.56	8.55	200
6	13	-18.37	-8.08	18.14	7.62	20.07	200
7	8	0.00	20.82	0.00	-21.51	20.82	200
7	9	-29.55	-6.46	29.55	5.55	30.24	200
9	10	-5.69	-2.65	5.68	2.62	6.28	200
9	14	-9.90	-2.62	9.78	2.36	10.24	200
10	11	3.68	3.18	-3.70	-3.23	4.87	200
12	13	-1.69	-0.96	1.68	0.95	1.94	200
13	14	-5.78	-2.77	5.72	2.64	6.41	200

Case 2: Real and Reactive power cost minimization with optimized tap settings

Table A1.9: LSF = 1.08

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0	50.003	-0.564	**LSF = 1.08**
2	1.045	-5.489	53.460	0.742	
3	1.010	-13.825	58.006	0.462	
4	1.027	-11.374	56.563	0.433	
5	1.033	-9.776	55.546	0.471	
6	1.070	-15.968	55.821	0.955	
7	1.055	-14.768	56.566	0.656	
8	1.090	-14.768	56.566	0.453	
9	1.048	-16.517	56.581	0.902	
10	1.044	-16.735	56.801	1.127	
11	1.053	-16.492	56.511	1.149	
12	1.054	-16.892	56.824	1.235	
13	1.048	-16.963	57.132	1.420	
14	1.029	-17.817	58.142	1.572	

Objective function:

Minimize Cost of Real Power

Value of objective function = 168.5913

Total Line Losses:

Real power loss = 0.158908 pu

Reactive power loss = 0.7380414 pu

Table A1.10: Generation Detail

Bus Number	Real Power	Reactive Power
1	2.556	-0.268
2	0.400	0.342
3		0.223
6		0.420
8		0.219

Table A1.11: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	1.100
9	4	9	0.9	1.1	0.969	1.001
10	5	6	0.9	1.1	0.962	0.900

Table A1.12: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-172.26	23.93	167.07	-39.70	173.92	200
1	5	-83.35	2.91	80.01	-16.56	83.40	200
2	3	-78.62	-3.06	75.95	-8.04	78.68	200
2	4	-60.41	8.10	58.45	-13.99	60.95	200
2	5	-44.61	7.45	43.55	-10.62	45.22	200
3	4	25.79	0.24	-26.23	-1.38	25.79	200
4	5	67.45	-9.01	-68.03	7.16	68.05	200
4	7	-27.90	53.11	27.90	-61.62	59.99	200
4	9	-17.35	3.17	17.35	-4.81	17.64	200
5	6	-52.57	-38.18	52.57	30.10	64.97	200
6	11	-7.70	-5.27	7.63	5.12	9.33	200
6	12	-8.42	-2.74	8.34	2.56	8.86	200
6	13	-19.10	-8.16	18.85	7.67	20.77	200
7	8	0.00	21.20	0.00	-21.91	21.20	200
7	9	-30.68	-6.56	30.68	5.59	31.37	200
9	10	-5.90	-2.56	5.89	2.53	6.44	200
9	14	-10.28	-2.59	10.15	2.31	10.60	200
10	11	3.83	3.27	-3.85	-3.32	5.04	200
12	13	-1.75	-0.96	1.74	0.96	2.00	200
13	14	-6.01	-2.83	5.94	2.69	6.64	200

Case 3: Real and Reactive Power cost minimization with optimized tap setting (Inv Tanh)

Table A1.13: LSF = 1.16

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0	50.003	-0.636	**LSF = 1.16**
2	1.045	-6.033	53.875	0.935	
3	1.010	-15.050	58.879	0.563	
4	1.025	-12.341	57.256	0.586	
5	1.031	-10.608	56.126	0.623	
6	1.070	-17.283	56.442	1.006	
7	1.053	-15.999	57.276	0.748	
8	1.090	-15.999	57.276	0.471	
9	1.047	-17.882	57.301	1.010	
10	1.043	-18.123	57.537	1.231	
11	1.053	-17.857	57.209	1.230	
12	1.053	-18.286	57.538	1.299	
13	1.047	-18.369	57.879	1.494	
14	1.027	-19.300	59.001	1.679	

Objective function:

Minimize Cost of Real Power

Value of objective function = 180.582

Total Line Losses:

Real power loss = 0.186589 pu

Reactive power loss = 0.807418 pu

Table A1.14: Generation Details

Bus Number	Real power	Reactive power
1	2.791	-0.299
2	0.400	0.413
3		0.268
6		0.438
8		0.228

Table A1.15: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	1.059
9	4	9	0.9	1.1	0.969	1.100
10	5	6	0.9	1.1	0.962	0.965

Table A1.16: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-188.79	27.55	182.54	-46.55	190.79	200
1	5	-90.31	2.33	86.39	-18.36	90.34	200
2	3	-84.76	-2.56	81.66	-10.35	84.80	200
2	4	-64.84	7.77	62.58	-14.54	65.30	200
2	5	-47.76	6.95	46.56	-10.57	48.27	200
3	4	27.61	-1.91	-28.12	0.60	27.68	200
4	5	72.57	-10.67	-73.25	8.52	73.35	200
4	7	-31.12	38.47	31.12	-43.86	49.48	200
4	9	-16.94	18.41	16.94	-22.42	25.02	200
5	6	-52.71	-2.04	52.71	-4.09	52.75	200
6	11	-8.29	-5.48	8.21	5.31	9.94	200
6	12	-9.05	-2.78	8.95	2.58	9.47	200
6	13	-20.55	-8.34	20.27	7.78	22.18	200
7	8	0.00	21.98	0.00	-22.75	21.98	200
7	9	-32.95	-6.78	32.95	5.66	33.64	200
9	10	-6.32	-2.38	6.31	2.34	6.76	200
9	14	-11.04	-2.53	10.89	2.21	11.32	200
10	11	4.13	3.46	-4.15	-3.51	5.38	200
12	13	-1.88	-0.98	1.87	0.97	2.12	200
13	14	-6.48	-2.95	6.40	2.79	7.12	200

RESULTS FOR 14 BUS SYSTEM (WITH OPTIMIZED TAP SETTINGS AND LOAD SCALING FACTOR)

Case 1: Real and Reactive power cost minimization with additional reactive support at load bus

Table A3.1: LSF =1.04

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.002	-0.530	**LSF = 1.04**
2	1.045	-5.217	53.256	0.646	
3	1.010	-13.213	57.575	0.409	
4	1.029	-10.899	56.210	0.310	
5	1.034	-9.363	55.245	0.354	
6	1.070	-15.283	55.471	0.787	
7	1.057	-14.168	56.201	0.533	
8	1.090	-14.168	56.201	0.425	
9	1.052	-15.849	56.209	0.718	
10	1.047	-16.049	56.414	0.939	
11	1.055	-15.801	56.134	0.966	
12	1.057	-16.200	56.419	0.990	
13	1.053	-16.350	56.709	1.000	
14	1.039	-17.240	57.652	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 162.3994

Total Line Losses:

Real power loss = 0.145421 pu
Reactive power loss = 0.671856 pu

Table A3.2:Optimized Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.900

Table A3.3: Generation Details

Bus No	Real Power Generation (pu)	Reactive Power Generation (pu)
1	2.439	-0.253
2	0.400	0.303
3		0.199
6		0.359
8		0.206

Table A3.4: Optimized Reactive support

Bus Number	Reactive Power Support (pu)
13	0.033
14	0.034

Table A3.5: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-164.04	22.06	159.33	-36.34	165.52	200
1	5	-79.86	3.28	76.80	-15.81	79.93	200
2	3	-75.55	-3.34	73.08	-6.91	75.62	200
2	4	-58.22	8.48	56.39	-13.97	58.83	200
2	5	-43.00	7.83	42.02	-10.80	43.71	200
3	4	24.89	1.55	-25.30	-2.61	24.94	200
4	5	65.06	-8.59	-65.60	6.88	65.63	200
4	7	-32.94	-48.25	32.94	42.79	58.43	200
4	9	-18.66	-19.60	18.66	16.48	27.06	200
5	6	-50.34	-38.66	50.34	30.97	63.47	200
6	11	-7.32	-4.57	7.26	4.45	8.63	200
6	12	-7.95	-1.81	7.87	1.66	8.15	200
6	13	-18.39	-4.47	18.19	4.06	18.93	200
7	8	0.00	20.01	0.00	-20.64	20.01	200
7	9	-29.65	-5.13	29.65	4.24	30.09	200
9	10	-5.77	-3.22	5.76	3.19	6.61	200
9	14	-9.99	-0.38	9.88	0.14	10.00	200
10	11	3.60	2.61	-3.62	-2.65	4.45	200
12	13	-1.53	-0.06	1.53	0.06	1.53	200
13	14	-5.67	-1.60	5.62	1.50	5.89	200

Case 2: Real and Reactive power cost minimization with additional reactive support at load bus

Table A3.6: LSF = 1.08

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.003	-0.567	**LSF = 1.08**
2	1.045	-5.487	53.455	0.730	
3	1.010	-13.821	57.995	0.456	
4	1.028	-11.380	56.540	0.373	
5	1.033	-9.775	55.522	0.416	
6	1.070	-15.932	55.763	0.798	
7	1.056	-14.780	56.539	0.567	
8	1.090	-14.780	56.539	0.432	
9	1.051	-16.527	56.549	0.754	
10	1.047	-16.738	56.762	0.972	
11	1.055	-16.477	56.463	0.990	
12	1.056	-16.892	56.754	1.000	
13	1.053	-17.055	57.059	1.000	
14	1.039	-17.989	58.055	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 168.2731

Total Line Losses:

Real power loss = 0.15825614 pu
Reactive power loss = 0.7220913 pu

Table A3.7: Optimized Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.900

Table A3.8: Generation Details

Bus No	Real Power Generation (pu)	Reactive Power Generation (pu)
1	2.555	-0.270
2	0.400	0.337
3		0.220
6		0.363
8		0.210

Table A3.9: Optimized Reactive support

Bus Number	Reactive Power Support (pu)
12	5.86E-6
13	0.035
14	0.037

Table A3.10: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-172.22	23.92	167.03	-39.68	173.87	200
1	5	-83.33	3.06	79.99	-16.70	83.39	200
2	3	-78.60	-3.06	75.93	-8.03	78.66	200
2	4	-60.42	8.37	58.46	-14.27	61.00	200
2	5	-44.57	7.63	43.52	-10.80	45.22	200
3	4	25.81	0.52	-26.25	-1.66	25.81	200
4	5	67.64	-9.44	-68.23	7.58	68.30	200
4	7	-34.21	-47.94	34.21	42.38	58.90	200
4	9	-19.37	-19.51	19.37	16.29	27.50	200
5	6	-52.29	-38.32	52.29	30.28	64.83	200
6	11	-7.61	-4.62	7.54	4.49	8.90	200
6	12	-8.24	-1.75	8.17	1.60	8.43	200
6	13	-19.12	-4.27	18.90	3.83	19.59	200
7	8	0.00	20.32	0.00	-20.97	20.32	200
7	9	-30.79	-5.12	30.79	4.16	31.22	200
9	10	-5.99	-3.19	5.97	3.15	6.78	200
9	14	-10.38	-0.16	10.26	-0.10	10.38	200
10	11	3.75	2.65	-3.76	-2.69	4.59	200
12	13	-1.58	0.00	1.58	-0.01	1.58	200
13	14	-5.89	-1.55	5.83	1.44	6.09	200

Case 3: Real and Reactive power cost minimization with additional reactive support at load bus

Table A3.11: LSF = 1.12

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.003	-0.604	**LSF = 1.12**
2	1.045	-5.758	53.658	0.820	
3	1.010	-14.431	58.424	0.504	
4	1.027	-11.863	56.878	0.439	
5	1.032	-10.190	55.805	0.482	
6	1.070	-16.584	56.061	0.805	
7	1.056	-15.395	56.884	0.601	
8	1.090	-15.395	56.884	0.439	
9	1.051	-17.209	56.898	0.789	
10	1.047	-17.432	57.118	1.000	
11	1.055	-17.161	56.799	1.000	
12	1.056	-17.589	57.096	1.000	
13	1.053	-17.760	57.417	1.000	
14	1.038	-18.740	58.466	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 174.4814

Total Line Losses:

Real power loss = 0.1717384 pu
Reactive power loss = 0.791743 pu

Table A3.12: Optimized Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	1.100
9	4	9	0.9	1.1	0.969	0.997
10	5	6	0.9	1.1	0.962	0.900

Table A3.13: Generation Details

Bus No	Real Power Generation (pu)	Reactive Power Generation (pu)
1	2.673	-0.285
2	0.400	0.372
3		0.242
6		0.366
8		0.213

Table A3.14: Optimized Reactive support

Bus Number	Reactive Power Support (pu)
10	4.782E-4
11	0.002
12	8.94E-4
13	0.037
14	0.040

Table A3.15: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-180.45	25.74	174.74	-43.08	182.27	200
1	5	-86.81	2.80	83.18	-17.61	86.85	200
2	3	-81.66	-2.80	78.78	-9.18	81.71	200
2	4	-62.63	8.25	60.53	-14.59	63.18	200
2	5	-46.14	7.41	45.02	-10.80	46.73	200
3	4	26.73	-0.52	-27.20	-0.71	26.73	200
4	5	70.23	-10.32	-70.87	8.30	70.98	200
4	7	-29.04	53.64	29.04	-62.57	60.99	200
4	9	-18.13	3.12	18.13	-4.89	18.40	200
5	6	-54.24	-37.98	54.24	29.58	66.22	200
6	11	-7.89	-4.54	7.82	4.40	9.10	200
6	12	-8.55	-1.66	8.47	1.49	8.71	200
6	13	-19.83	-4.09	19.60	3.63	20.25	200
7	8	0.00	20.60	0.00	-21.28	20.60	200
7	9	-31.94	-5.07	31.94	4.03	32.34	200
9	10	-6.21	-3.06	6.20	3.02	6.92	200
9	14	-10.77	0.04	10.64	-0.32	10.77	200
10	11	3.88	2.73	-3.90	-2.77	4.75	200
12	13	-1.64	0.02	1.63	-0.02	1.64	200
13	14	-6.11	-1.49	6.05	1.36	6.29	200

Case 4: Real and Reactive power cost minimization with additional reactive support at load bus

Table A3.16: LSF = 1.16

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.003	-4.908	**LSF = 1.16**
2	1.045	-6.031	54.712	0.923	
3	1.010	-15.046	59.670	0.557	
4	1.026	-12.346	57.743	0.032	
5	1.031	-10.606	56.496	-0.244	
6	1.070	-17.249	56.756	0.832	
7	1.055	-16.008	57.670	0.441	
8	1.090	-16.008	57.670	0.453	
9	1.049	-17.889	57.653	0.645	
10	1.045	-18.123	57.878	0.894	
11	1.054	-17.842	57.535	0.970	
12	1.056	-18.302	57.846	1.000	
13	1.053	-18.473	58.182	1.000	
14	1.035	-19.452	59.294	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 180.134

Total Line Losses:

Real power loss = 0.1859018 pu
Reactive power loss = 0.830725 pu

Table A3.17: Optimized Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.900
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.900

Table A3.18: Generation Details

Bus No	Real Power Generation (pu)	Reactive Power Generation (pu)
1	2.790	-0.300
2	0.400	0.409
3		0.265
6		0.376
8		0.219

Table A3.19: Optimized Reactive Support

Bus Number	Reactive Power Support (pu)
12	0.003
13	0.042
14	0.030

Table A3.20: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-188.74	27.54	182.49	-46.53	190.74	200
1	5	-90.29	2.46	86.37	-18.49	90.33	200
2	3	-84.74	-2.56	81.64	-10.35	84.78	200
2	4	-64.85	8.02	62.59	-14.80	65.34	200
2	5	-47.73	7.11	46.53	-10.73	48.26	200
3	4	27.63	-1.66	-28.14	0.35	27.68	200
4	5	72.74	-11.06	-73.43	8.89	73.58	200
4	7	-36.72	-47.44	36.72	41.64	59.99	200
4	9	-20.78	-19.46	20.78	15.99	28.47	200
5	6	-56.28	-37.59	56.28	28.79	67.68	200
6	11	-8.21	-4.90	8.13	4.74	9.56	200
6	12	-8.88	-1.58	8.80	1.39	9.02	200
6	13	-20.57	-4.12	20.32	3.62	20.98	200
7	8	0.00	21.19	0.00	-21.90	21.19	200
7	9	-33.05	-5.49	33.05	4.38	33.50	200
9	10	-6.40	-2.94	6.39	2.90	7.04	200
9	14	-11.13	-0.35	10.98	0.04	11.13	200
10	11	4.05	2.90	-4.07	-2.94	4.98	200
12	13	-1.72	-0.06	1.71	0.05	1.72	200
13	14	-6.37	-2.08	6.30	1.93	6.70	200

Comparisons of Bus Marginal costs (Real & Reactive) with varying LSF

(No Reactive Support at Load Bus)

Bus Number								
1	50.002	-0.489	50.002	-0.527	50.003	-0.564	50.003	-0.636
2	53.065	0.578	53.26	0.657	53.46	0.742	53.875	0.935
3	57.172	0.368	57.584	0.414	58.006	0.462	58.879	0.563
4	55.904	0.3	56.23	0.364	56.563	0.433	57.256	0.586
5	54.994	0.342	55.267	0.404	55.546	0.471	56.126	0.623
6	55.233	0.908	55.523	0.931	55.821	0.955	56.442	1.006
7	55.892	0.575	56.225	0.614	56.566	0.656	57.276	0.748
8	55.892	0.435	56.225	0.444	56.566	0.453	57.276	0.471
9	55.9	0.806	56.236	0.853	56.581	0.902	57.301	1.01
10	56.104	1.036	56.448	1.08	56.801	1.127	57.537	1.231
11	55.851	1.076	56.177	1.112	56.511	1.149	57.209	1.23
12	56.147	1.178	56.482	1.206	56.824	1.235	57.538	1.299
13	56.424	1.354	56.774	1.386	57.132	1.42	57.879	1.494
14	57.327	1.476	57.729	1.522	58.142	1.572	59.001	1.679

Comparisons of Bus Marginal costs (Real & Reactive) with varying LSF

(With Reactive Support at Load Bus)

Bus Number								
1	50.002	-0.491	50.002	-0.53	50.003	-0.567	50.003	-4.908
2	53.061	0.568	53.256	0.646	53.455	0.73	54.712	0.923
3	57.164	0.363	57.575	0.409	57.995	0.456	59.67	0.557
4	55.886	0.251	56.21	0.31	56.54	0.373	57.743	0.032
5	54.974	0.296	55.245	0.354	55.522	0.416	56.496	-0.244
6	55.186	0.777	55.471	0.787	55.763	0.798	56.756	0.832
7	55.872	0.501	56.201	0.533	56.539	0.567	57.67	0.441
8	55.872	0.418	56.201	0.425	56.539	0.432	57.67	0.453
9	55.876	0.684	56.209	0.718	56.549	0.754	57.653	0.645
10	56.074	0.907	56.414	0.939	56.762	0.972	57.878	0.894
11	55.813	0.943	56.134	0.966	56.463	0.99	57.535	0.97
12	56.092	0.98	56.419	0.99	56.754	1	57.846	1
13	56.366	1	56.709	1	57.059	1	58.182	1
14	57.259	1	57.652	1	58.055	1	59.294	1

Reactive Support Provided At Buses

Bus Number	Load Scale Factor 1.00	Load Scale Factor 1.04	Load Scale Factor 1.08	Load Scale Factor 1.16
12			5.86 E -6	0.003
13	0.031	0.033	0.035	0.042
14	0.031	0.034	0.035	0.030

RESULTS FOR 14 BUS SYSTEM

WITH OPTIMIZED TAP SETTINGS AND LOAD SCALING FACTOR

Case: Real and Reactive power cost minimization with additional reactive support at load bus

Table A2. 1: Reactive Power cost modeled as Inverse Tanh x

Bus Number	Voltage (p.u.)	Angle (Degrees)	Bus Marginal Cost (Real)	Bus Marginal Cost (Reactive)	Remarks
1	1.060	0.000	50.003	-0.630	**LSF = 1.15**
2	1.045	-5.963	53.816	0.898	
3	1.010	-14.892	58.756	0.544	
4	1.026	-12.224	57.145	0.512	
5	1.031	-10.502	56.030	0.553	
6	1.070	-17.086	56.309	0.859	
7	1.055	-15.853	57.159	0.658	
8	1.090	-15.853	57.159	0.451	
9	1.050	-17.718	57.178	0.865	
10	1.045	-17.949	57.405	1.080	
11	1.054	-17.672	57.074	1.077	
12	1.055	-18.110	57.382	1.081	
13	1.052	-18.271	57.715	1.111	
14	1.035	-19.259	58.809	1.165	

Objective function:

Minimize Cost of Real Power

Value of objective function = 179.1182

Total Line Losses:

Real power loss = 0.182364 pu

Reactive power loss = 0.828973 pu

Table A2. 2: Specified Tap Setting

Line No	From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
			Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
8	4	7	0.9	1.1	0.978	0.943
9	4	9	0.9	1.1	0.969	0.900
10	5	6	0.9	1.1	0.962	0.900

Table A2. 3: Optimized Capacitor Placement at load buses.

Bus Number	Capacitance (pu MVAR)
13	0.031
14	0.031

Table A2. 4: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	2.761	-0.296	
2	0.400	0.400	
3		0.260	
6		0.386	
8		0.218	

Table A2. 5: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-186.66	27.09	180.55	-45.67	188.62	200
1	5	-89.42	2.53	85.58	-18.26	89.46	200
2	3	-83.97	-2.62	80.93	-10.05	84.01	200
2	4	-64.29	8.05	62.07	-14.72	64.79	200
2	5	-47.34	7.17	46.15	-10.73	47.88	200
3	4	27.40	-1.40	-27.90	0.11	27.44	200
4	5	72.10	-10.83	-72.77	8.70	72.91	200
4	7	-29.78	53.45	29.78	-62.51	61.19	200
4	9	-16.85	18.80	16.85	-22.87	25.25	200
5	6	-51.53	1.98	51.53	-7.97	51.57	200
6	11	-8.14	-4.90	8.06	4.74	9.50	200
6	12	-8.82	-1.92	8.73	1.74	9.02	200
6	13	-20.38	-4.94	20.13	4.44	20.97	200
7	8	0.00	21.13	0.00	-21.84	21.13	200
7	9	-32.76	-5.52	32.76	4.43	33.22	200
9	10	-6.34	-2.94	6.33	2.90	6.99	200
9	14	-11.03	-0.47	10.89	0.17	11.04	200
10	11	4.02	2.90	-4.04	-2.94	4.96	200
12	13	-1.71	-0.14	1.71	0.13	1.72	200
13	14	-6.31	-1.87	6.25	1.73	6.58	200

APPENDIX B

Table B.1: Line Data for IEEE 30 Bus System

Line from	Line To	Resistance	Reactance	Line Charging Capacitance	Tap Positions
1	2	0.0192	0.0575	0.0528	
1	3	0.0452	0.1652	0.0408	
2	4	0.057	0.1737	0.0368	
3	4	0.0472	0.0379	0.0084	
2	5	0.0581	0.1983	0.0418	
2	6	0.0132	0.1763	0.0374	
4	6	0.0119	0.0414	0.009	
5	7	0.046	0.116	0.0204	
6	7	0.0267	0.082	0.017	
6	8	0.012	0.042	0.009	
6	9	0	0.208		0.978
6	10	0	0.556		0.969
9	11	0	0.208		
9	10	0	0.11		-
4	12	0	0.256		0.932
12	13	0	0.14		
12	14	0.1231	0.2559		
12	15	0.0662	0.1304		
12	16	0.0945	0.1987		
14	15	0.221	0.1997		
16	17	0.0524	0.1923		
15	18	0.1073	0.2185		
18	19	0.0639	0.1292		
19	20	0.034	0.068		
10	20	0.0936	0.209		
10	17	0.0324	0.0845		
10	21	0.0348	0.0749		
10	22	0.0727	0.1499		
21	22	0.0116	0.0236		
15	23	0.1	0.202		
22	24	0.115	0.179		
23	24	0.132	0.27		
24	25	0.1885	0.3292		
25	26	0.2544	0.38		
25	27	0.1093	0.2087		
28	27	0	0.396		0.968
27	29	0.2198	0.4153		
27	30	0.3202	0.6027		
29	30	0.2399	0.4533		
8	28	0.0636	0.2	0.0428	
6	28	0.0169	0.0599	0.013	

Table B.2: Bus Data for IEEE 30 Bus System

Bus Number	Generation		Load		Voltage Specified	Capacitors
	Real Power	Reactive Power	Real	Reactive		
1	2.602	-0.161	0	0	1.06	
2	0.5856	0.5	0.217	0.127	1.0338	
3	0	0	0.024	0.012		
4	0	0	0.076	0.016		
5	0.2456	0.37	0.942	0.19	1.0058	
6	0	0	0	0		
7	0	0	0.228	0.109		
8	0.35	0.373	0.3	0.3	1.023	
9	0	0	0	0		
10	0	0	0.058	0.02		0.19
11	0.1793	0.162	0	0	1.0913	
12	0	0	0.112	0.075		
13	0.1691	0.106	0	0	1.0883	
14	0	0	0.062	0.016		
15	0	0	0.082	0.025		
16	0	0	0.035	0.018		
17	0	0	0.09	0.058		
18	0	0	0.032	0.009		
19	0	0	0.095	0.034		
20	0	0	0.022	0.007		
21	0	0	0.175	0.112		
22	0	0	0	0		
23	0	0	0.032	0.016		
24	0	0	0.087	0.067		0.043
25	0	0	0	0		
26	0	0	0.035	0.023		
27	0	0	0	0		
28	0	0	0	0		
29	0	0	0.024	0.009		
30	0	0	0.106	0.019		

Base MVA = 100

RESULTS OF 30 BUS SYSTEM

Case –1: Real Power Cost Minimization with Standard tap settings and optimized reactive support at load bus.

Table B.1

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.010	EPS	
2	1.034	-2.620	2.071	EPS	
3	1.032	-4.422	2.148	0.128	
4	1.028	-5.406	2.189	0.155	
5	1.006	-8.825	2.201	EPS	
6	1.023	-6.296	2.181	0.087	
7	1.008	-7.861	2.207	0.058	
8	1.023	-6.262	2.155	EPS	
9	1.048	-7.981	2.171	0.227	
10	1.038	-9.898	2.183	0.427	
11	1.091	-6.113	2.164	EPS	
12	1.050	-9.150	2.321	1.000	
13	1.088	-7.963	2.265	1.735	
14	1.035	-10.063	2.382	0.945	
15	1.030	-10.163	2.354	0.883	
16	1.037	-9.734	2.267	0.775	
17	1.033	-10.065	2.229	0.542	
18	1.021	-10.774	2.344	0.741	
19	1.018	-10.943	2.322	0.652	
20	1.023	-10.740	2.287	0.597	
21	1.026	-10.373	2.222	0.463	
22	1.026	-10.367	2.223	0.466	
23	1.019	-10.601	2.345	0.748	
24	1.012	-10.838	2.299	0.548	
25	1.003	-10.669	2.240	0.404	
26	0.985	-11.101	2.292	0.439	
27	1.006	-10.301	2.184	0.295	
28	1.021	-6.691	2.189	0.092	
29	0.986	-11.575	2.261	0.317	
30	0.974	-12.490	2.315	0.326	

Objective function:

Minimize Cost of Real Power

Value of objective function = 6.4866

Table B.2: Transformer Tap Settings

From Bus	To Bus	Tap Limits		Specified Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)	
6	9	0.9	1.1	0.978
6	10	0.9	1.1	0.969
4	12	0.9	1.1	0.932
28	27	0.9	1.1	0.968

Table B.3: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.376	-0.033	
2	0.586	-0.006	
5	0.246	0.212	
8	0.350	0.259	
11	0.179	0.232	
13	0.169	0.300	

Table B.4: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-87.14	0.45	85.82	-4.33	87.15	200
1	3	-50.44	2.89	49.40	-6.62	50.52	200
2	4	-28.15	6.80	27.72	-8.10	28.96	200
2	5	-57.64	-1.72	46.72	-4.26	57.66	200
2	6	-36.89	6.86	56.16	-4.35	37.53	200
3	4	-47.00	3.48	36.14	-9.10	47.13	200
4	6	-39.30	1.14	39.13	-1.74	39.32	200
4	12	-29.54	-23.65	-13.56	1.87	37.84	200
5	7	13.48	-2.10	36.36	5.22	13.64	200
6	7	-36.72	-6.29	-1.08	0.95	37.26	200
6	8	1.08	-0.95	15.51	-0.81	1.44	200
6	9	-15.51	0.36	12.40	2.92	15.51	200
6	10	-12.40	-3.76	33.10	8.11	12.96	200
6	28	-12.45	-0.83	-17.93	-23.20	12.47	200
8	28	-3.92	2.21	29.54	20.63	4.50	200
9	10	-33.10	-9.29	-16.91	-30.00	34.38	200
9	11	17.93	21.70	7.86	2.26	28.15	200
10	17	-5.43	-4.52	17.94	6.46	7.07	200
10	20	-9.17	-3.80	1.65	0.65	9.92	200
10	21	-16.58	-10.31	7.09	3.15	19.52	200
10	22	-8.14	-4.79	5.42	4.48	9.44	200
12	13	16.91	28.60	3.58	1.32	33.22	200
12	14	-7.93	-2.42	5.84	1.44	8.29	200
12	15	-18.17	-6.91	2.64	0.53	19.44	200
12	16	-7.14	-3.26	9.08	3.61	7.85	200
14	15	-1.66	-0.66	-6.88	-2.91	1.78	200
15	18	-5.88	-1.51	16.45	10.05	6.07	200
15	23	-5.52	-3.11	8.08	4.67	6.33	200
16	17	-3.59	-1.35	-1.05	-1.15	3.83	200
18	19	-2.64	-0.54	5.48	3.03	2.70	200
19	20	6.86	2.87	6.96	3.41	7.44	200
21	22	1.05	1.15	2.27	1.41	1.56	200
22	24	-7.03	-3.51	0.52	2.50	7.86	200
23	24	-2.28	-1.43	3.50	2.30	2.69	200
24	25	-0.53	-2.52	-3.04	0.11	2.58	200

25	26	-3.55	-2.37	12.42	0.74	4.26	200
25	27	3.03	-0.13	3.91	-2.23	3.03	200
27	28	16.87	11.79	-16.87	-13.45	20.58	200
27	29	-6.19	-1.68	6.11	1.51	6.42	200
27	30	-7.10	-1.67	6.93	1.36	7.29	200
29	30	-3.71	-0.61	3.67	0.54	3.75	200

<u>Case 2:</u> Real Power Cost Minimization with optimized tap settings and addition of reactive support at load buses

<u>Table B.5: Taps at optimized settings</u>

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.010	EPS	
2	1.034	-2.620	2.071	EPS	
3	1.032	-4.422	2.148	0.128	
4	1.028	-5.406	2.189	0.155	
5	1.006	-8.825	2.201	EPS	
6	1.023	-6.296	2.181	0.087	
7	1.008	-7.861	2.207	0.058	
8	1.023	-6.262	2.155	EPS	
9	1.048	-7.981	2.171	0.227	
10	1.038	-9.898	2.183	0.427	
11	1.091	-6.113	2.164	EPS	
12	1.050	-9.150	2.321	1.000	
13	1.088	-7.963	2.265	1.735	
14	1.035	-10.063	2.382	0.945	
15	1.030	-10.163	2.354	0.883	
16	1.037	-9.734	2.267	0.775	
17	1.033	-10.065	2.229	0.542	
18	1.021	-10.774	2.344	0.741	
19	1.018	-10.943	2.322	0.652	
20	1.023	-10.740	2.287	0.597	
21	1.026	-10.373	2.222	0.463	
22	1.026	-10.367	2.223	0.466	
23	1.019	-10.601	2.345	0.748	
24	1.012	-10.838	2.299	0.548	
25	1.003	-10.669	2.240	0.404	
26	0.985	-11.101	2.292	0.439	
27	1.006	-10.301	2.184	0.295	
28	1.021	-6.691	2.189	0.092	
29	0.986	-11.575	2.261	0.317	
30	0.974	-12.490	2.315	0.326	

Objective function:

Minimize Cost of Real Power

Value of objective function = 6.3028

Table B.6: Transformer Tap Setting

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.900
4	12	0.9	1.1	0.932	0.900
28	27	0.9	1.1	0.968	0.963

Table B.7: Placement of Capacitors at load buses

Bus Number	Capacitance (pu MVAR)
12	0.015

Table B.8: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.376	-0.033	
2	0.586	-0.006	
5	0.246	0.212	
8	0.350	0.259	
11	0.179	0.232	
13	0.169	0.300	

Table B.9: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-87.14	0.45	85.82	-4.33	87.15	200
1	3	-50.44	2.89	49.40	-6.62	50.52	200
2	4	-28.15	6.80	27.72	-8.10	28.96	200
2	5	-57.64	-1.72	46.72	-4.26	57.66	200
2	6	-36.89	6.86	56.16	-4.35	37.53	200
3	4	-47.00	3.48	36.14	-9.10	47.13	200
4	6	-39.30	1.14	39.13	-1.74	39.32	200
4	12	-30.60	-41.97	-13.56	1.87	51.94	200
5	7	13.48	-2.10	36.36	5.22	13.64	200
6	7	-36.72	-6.29	-1.08	0.95	37.26	200
6	8	1.08	-0.95	16.85	44.85	1.44	200
6	9	-16.85	-49.20	13.35	18.02	52.01	200
6	10	-13.35	-20.61	33.10	8.11	24.56	200
6	28	-12.45	-0.83	-17.93	-23.20	12.47	200
8	28	-3.92	2.21	30.60	36.68	4.50	200
9	10	-33.10	-9.29	-16.91	-30.00	34.38	200
9	11	17.93	21.70	7.86	2.26	28.15	200
10	17	-5.43	-4.52	17.94	6.46	7.07	200
10	20	-9.17	-3.80	1.65	0.65	9.92	200
10	21	-16.58	-10.31	7.09	3.15	19.52	200
10	22	-8.14	-4.79	5.42	4.48	9.44	200
12	13	16.91	28.60	3.58	1.32	33.22	200
12	14	-7.93	-2.42	5.84	1.44	8.29	200
12	15	-18.17	-6.91	2.64	0.53	19.44	200
12	16	-7.14	-3.26	9.08	3.61	7.85	200
14	15	-1.66	-0.66	-6.88	-2.91	1.78	200
15	18	-5.88	-1.51	16.45	10.05	6.07	200
15	23	-5.52	-3.11	8.08	4.67	6.33	200
16	17	-3.59	-1.35	-1.05	-1.15	3.83	200
18	19	-2.64	-0.54	5.48	3.03	2.70	200
19	20	6.86	2.87	6.96	3.41	7.44	200
21	22	1.05	1.15	2.27	1.41	1.56	200
22	24	-7.03	-3.51	0.52	2.50	7.86	200
23	24	-2.28	-1.43	3.50	2.30	2.69	200
24	25	-0.53	-2.52	-3.04	0.11	2.58	200
25	26	-3.55	-2.37	12.42	0.74	4.26	200

25	27	3.03	-0.13	3.91	-2.23	3.03	200
27	28	16.95	13.13	-16.95	-14.93	21.44	200
27	29	-6.19	-1.68	6.11	1.51	6.42	200
27	30	-7.10	-1.67	6.93	1.36	7.29	200
29	30	-3.71	-0.61	3.67	0.54	3.75	200

Case 3: **Real and Reactive Power cost minimization with optimized tap setting and optimized reactive support at load buses.**

Table B.10: Reactive power modeled as Sinh function

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.010	-0.067	
2	1.034	-2.620	2.087	-0.011	
3	1.032	-4.422	2.216	0.284	
4	1.028	-5.406	2.282	0.361	
5	1.006	-8.825	2.393	0.431	
6	1.023	-6.296	2.326	0.431	
7	1.008	-7.861	2.378	0.443	
8	1.023	-6.262	2.352	0.529	
9	1.048	-7.981	2.340	0.567	
10	1.038	-9.898	2.374	0.689	
11	1.091	-6.113	2.306	0.472	
12	1.050	-9.150	2.412	1.000	
13	1.088	-7.963	2.361	1.470	
14	1.035	-10.063	2.483	0.981	
15	1.030	-10.163	2.478	0.952	
16	1.037	-9.734	2.412	0.886	
17	1.033	-10.065	2.405	0.759	
18	1.021	-10.774	2.499	0.880	
19	1.018	-10.943	2.495	0.832	
20	1.023	-10.740	2.466	0.798	
21	1.026	-10.373	2.419	0.723	
22	1.026	-10.367	2.419	0.724	
23	1.019	-10.601	2.498	0.888	
24	1.012	-10.838	2.488	0.783	
25	1.003	-10.669	2.451	0.702	
26	0.985	-11.101	2.516	0.746	
27	1.006	-10.301	2.401	0.629	
28	1.021	-6.691	2.351	0.473	
29	0.986	-11.575	2.502	0.658	
30	0.974	-12.490	2.573	0.670	

Objective function:

Minimize Cost of Real Power

Value of objective function = 6.56282

Table B.11: Tap Setting

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.900
4	12	0.9	1.1	0.932	0.900
28	27	0.9	1.1	0.968	0.963

Table B.12: Optimal Capacitor Placement at load buses

Bus Number	Capacitance (pu MVAR)
12	0.015

Table B.13: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.376	-0.033	
2	0.586	-0.006	
5	0.246	0.212	
8	0.350	0.259	
11	0.179	0.232	
13	0.169	0.300	

Table B.14: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-87.14	0.45	85.82	-4.33	87.15	200
1	3	-50.44	2.89	49.40	-6.62	50.52	200
2	4	-28.15	6.80	27.72	-8.10	28.96	200
2	5	-57.64	-1.72	46.72	-4.26	57.66	200
2	6	-36.89	6.86	56.16	-4.35	37.53	200
3	4	-47.00	3.48	36.14	-9.10	47.13	200
4	6	-39.30	1.14	39.13	-1.74	39.32	200
4	12	-30.60	-41.97	-13.56	1.87	51.94	200
5	7	13.48	-2.10	36.36	5.22	13.64	200
6	7	-36.72	-6.29	-1.08	0.95	37.26	200
6	8	1.08	-0.95	16.85	44.85	1.44	200
6	9	-16.85	-49.20	13.35	18.02	52.01	200
6	10	-13.35	-20.61	33.10	8.11	24.56	200
6	28	-12.45	-0.83	-17.93	-23.20	12.47	200
8	28	-3.92	2.21	30.60	36.68	4.50	200
9	10	-33.10	-9.29	-16.91	-30.00	34.38	200
9	11	17.93	21.70	7.86	2.26	28.15	200
10	17	-5.43	-4.52	17.94	6.46	7.07	200
10	20	-9.17	-3.80	1.65	0.65	9.92	200
10	21	-16.58	-10.31	7.09	3.15	19.52	200

10	22	-8.14	-4.79	5.42	4.48	9.44	200
12	13	16.91	28.60	3.58	1.32	33.22	200
12	14	-7.93	-2.42	5.84	1.44	8.29	200
12	15	-18.17	-6.91	2.64	0.53	19.44	200
12	16	-7.14	-3.26	9.08	3.61	7.85	200
14	15	-1.66	-0.66	-6.88	-2.91	1.78	200
15	18	-5.88	-1.51	16.45	10.05	6.07	200
15	23	-5.52	-3.11	8.08	4.67	6.33	200
16	17	-3.59	-1.35	-1.05	-1.15	3.83	200
18	19	-2.64	-0.54	5.48	3.03	2.70	200
19	20	6.86	2.87	6.96	3.41	7.44	200
21	22	1.05	1.15	2.27	1.41	1.56	200
22	24	-7.03	-3.51	0.52	2.50	7.86	200
23	24	-2.28	-1.43	3.50	2.30	2.69	200
24	25	-0.53	-2.52	-3.04	0.11	2.58	200
25	26	-3.55	-2.37	12.42	0.74	4.26	200
25	27	3.03	-0.13	3.91	-2.23	3.03	200
27	28	16.95	13.13	-16.95	-14.93	21.44	200
27	29	-6.19	-1.68	6.11	1.51	6.42	200
27	30	-7.10	-1.67	6.93	1.36	7.29	200
29	30	-3.71	-0.61	3.67	0.54	3.75	200

Case 4: Real and Reactive Power cost minimization with optimized tap setting and optimized reactive support at load buses

Table B.15: Reactive power modeled as Inverse Tanh function

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.010	-0.067	
2	1.034	-2.620	2.087	-0.011	
3	1.032	-4.422	2.218	0.288	
4	1.028	-5.406	2.284	0.366	
5	1.006	-8.825	2.396	0.438	
6	1.023	-6.296	2.329	0.439	
7	1.008	-7.861	2.381	0.450	
8	1.023	-6.262	2.356	0.542	
9	1.048	-7.981	2.343	0.574	
10	1.038	-9.898	2.378	0.694	
11	1.091	-6.113	2.309	0.481	
12	1.050	-9.150	2.414	1.000	
13	1.088	-7.963	2.363	1.464	
14	1.035	-10.063	2.485	0.982	
15	1.030	-10.163	2.481	0.953	
16	1.037	-9.734	2.415	0.888	
17	1.033	-10.065	2.409	0.764	
18	1.021	-10.774	2.503	0.883	
19	1.018	-10.943	2.498	0.836	
20	1.023	-10.740	2.470	0.802	
21	1.026	-10.373	2.423	0.728	
22	1.026	-10.367	2.423	0.730	
23	1.019	-10.601	2.501	0.891	
24	1.012	-10.838	2.492	0.788	
25	1.003	-10.669	2.455	0.709	
26	0.985	-11.101	2.521	0.753	
27	1.006	-10.301	2.406	0.637	
28	1.021	-6.691	2.354	0.483	
29	0.986	-11.575	2.508	0.666	
30	0.974	-12.490	2.579	0.678	

Objective function:

Minimize Cost of Real Power

Value of objective function = 6.5659

Table B.16: Tap Setting

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.900
4	12	0.9	1.1	0.932	0.900
28	27	0.9	1.1	0.968	0.963

Table B.17: Optimized Capacitor Placement at load buses

Bus Number	Capacitance (pu MVAR)
12	0.015

Table B.18: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.376	-0.033	
2	0.586	-0.006	
5	0.246	0.212	
8	0.350	0.259	
11	0.179	0.232	
13	0.169	0.300	

Table B.19: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-87.14	0.45	85.82	-4.33	87.15	200
1	3	-50.44	2.89	49.40	-6.62	50.52	200
2	4	-28.15	6.80	27.72	-8.10	28.96	200
2	5	-57.64	-1.72	46.72	-4.26	57.66	200
2	6	-36.89	6.86	56.16	-4.35	37.53	200
3	4	-47.00	3.48	36.14	-9.10	47.13	200
4	6	-39.30	1.14	39.13	-1.74	39.32	200
4	12	-30.60	-41.97	-13.56	1.87	51.94	200
5	7	13.48	-2.10	36.36	5.22	13.64	200
6	7	-36.72	-6.29	-1.08	0.95	37.26	200
6	8	1.08	-0.95	16.85	44.85	1.44	200
6	9	-16.85	-49.20	13.35	18.02	52.01	200
6	10	-13.35	-20.61	33.10	8.11	24.56	200
6	28	-12.45	-0.83	-17.93	-23.20	12.47	200
8	28	-3.92	2.21	30.60	36.68	4.50	200
9	10	-33.10	-9.29	-16.91	-30.00	34.38	200
9	11	17.93	21.70	7.86	2.26	28.15	200
10	17	-5.43	-4.52	17.94	6.46	7.07	200
10	20	-9.17	-3.80	1.65	0.65	9.92	200
10	21	-16.58	-10.31	7.09	3.15	19.52	200

10	22	-8.14	-4.79	5.42	4.48	9.44	200
12	13	16.91	28.60	3.58	1.32	33.22	200
12	14	-7.93	-2.42	5.84	1.44	8.29	200
12	15	-18.17	-6.91	2.64	0.53	19.44	200
12	16	-7.14	-3.26	9.08	3.61	7.85	200
14	15	-1.66	-0.66	-6.88	-2.91	1.78	200
15	18	-5.88	-1.51	16.45	10.05	6.07	200
15	23	-5.52	-3.11	8.08	4.67	6.33	200
16	17	-3.59	-1.35	-1.05	-1.15	3.83	200
18	19	-2.64	-0.54	5.48	3.03	2.70	200
19	20	6.86	2.87	6.96	3.41	7.44	200
21	22	1.05	1.15	2.27	1.41	1.56	200
22	24	-7.03	-3.51	0.52	2.50	7.86	200
23	24	-2.28	-1.43	3.50	2.30	2.69	200
24	25	-0.53	-2.52	-3.04	0.11	2.58	200
25	26	-3.55	-2.37	12.42	0.74	4.26	200
25	27	3.03	-0.13	3.91	-2.23	3.03	200
27	28	16.95	13.13	-16.95	-14.93	21.44	200
27	29	-6.19	-1.68	6.11	1.51	6.42	200
27	30	-7.10	-1.67	6.93	1.36	7.29	200
29	30	-3.71	-0.61	3.67	0.54	3.75	200

APPENDIX B1

RESULTS OF 30 BUS SYSTEM WITH VARYING LOAD SCALING FACTOR (LSF)

Case –1: Real and Reactive Power Cost (inv. Tanh) Minimization with optimized tap settings and optimized Reactive support at load buses

Table B1.1: LSF = 1.04

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.011	-0.120	**LSF =1.04**
2	1.034	-2.899	2.131	0.050	
3	1.032	-4.774	2.250	0.306	
4	1.027	-5.840	2.328	0.399	
5	1.006	-9.480	2.465	0.490	
6	1.023	-6.808	2.383	0.487	
7	1.008	-8.451	2.443	0.502	
8	1.023	-6.813	2.416	0.604	
9	1.047	-8.617	2.402	0.599	
10	1.037	-10.602	2.440	0.719	
11	1.091	-6.746	2.366	0.483	
12	1.050	-9.841	2.464	1.000	
13	1.088	-8.654	2.414	1.434	
14	1.034	-10.790	2.540	0.987	
15	1.029	-10.889	2.539	0.962	
16	1.036	-10.439	2.473	0.900	
17	1.031	-10.779	2.471	0.786	
18	1.019	-11.522	2.567	0.899	
19	1.016	-11.696	2.564	0.856	
20	1.021	-11.483	2.535	0.823	
21	1.023	-11.096	2.488	0.755	
22	1.024	-11.089	2.488	0.757	
23	1.017	-11.339	2.565	0.909	
24	1.009	-11.577	2.560	0.817	
25	1.000	-11.398	2.526	0.749	
26	0.982	-11.850	2.597	0.797	
27	1.004	-11.013	2.475	0.682	
28	1.020	-7.228	2.411	0.533	
29	0.983	-12.344	2.587	0.714	
30	0.970	-13.302	2.664	0.727	

Objective function:

Minimize Cost of Real Power

Value of objective function = 7.04291

Total Line Losses:

Real power loss = 0.080396 pu
Reactive power loss = 0.43274 pu

Table B1.2: Transformer Tap Settings

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.900
4	12	0.9	1.1	0.932	0.940
28	27	0.9	1.1	0.968	1.100

Table B1.3: Optimized Capacitor Placement at load buses

Bus Number	Capacitance (pu MVAR)
12	0.034

Table B1.4: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.499	-0.060	
2	0.586	0.025	
5	0.246	0.240	
8	0.350	0.294	
11	0.179	0.237	
13	0.169	0.300	

Table B1.5: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-95.56	2.82	93.97	-7.49	95.60	200
1	3	-54.34	3.20	53.13	-7.55	54.43	200
2	4	-29.78	6.80	29.30	-8.25	30.55	200
2	5	-60.96	-1.31	50.31	-5.37	60.97	200
2	6	-39.22	7.09	59.31	-5.49	39.85	200
3	4	-50.63	4.45	38.37	-9.61	50.83	200
4	6	-42.32	2.29	42.14	-2.91	42.38	200
4	12	-32.66	-41.70	-14.19	2.75	52.97	200
5	7	14.10	-3.00	37.91	4.79	14.41	200
6	7	-38.28	-5.91	0.11	-0.93	38.73	200
6	8	-0.08	1.01	18.06	44.91	1.01	200
6	9	-18.06	-49.34	13.44	9.21	52.54	200
6	10	-13.44	-10.59	34.18	8.65	17.11	200
6	28	-13.18	-0.78	-17.93	-23.72	13.21	200
8	28	-3.91	1.83	32.66	36.19	4.31	200
9	10	-34.18	-9.93	-16.91	-30.00	35.60	200
9	11	17.93	22.17	8.19	2.41	28.52	200
10	17	-5.63	-4.46	18.68	6.95	7.18	200
10	20	-9.51	-3.83	1.73	0.74	10.25	200
10	21	-17.20	-10.71	7.39	3.52	20.26	200
10	22	-8.44	-4.97	5.62	4.42	9.79	200
12	13	16.91	28.60	3.74	1.62	33.22	200
12	14	-8.27	-2.58	6.10	1.62	8.67	200
12	15	-18.93	-7.43	2.77	0.68	20.34	200
12	16	-7.45	-3.64	9.42	3.63	8.29	200
14	15	-1.74	-0.74	-7.13	-2.90	1.89	200
15	18	-6.14	-1.71	17.07	10.43	6.38	200
15	23	-5.74	-3.37	8.37	4.84	6.66	200
16	17	-3.75	-1.65	-1.13	-1.22	4.10	200
18	19	-2.78	-0.69	5.70	3.29	2.86	200
19	20	7.11	2.86	7.17	3.50	7.66	200
21	22	1.13	1.22	2.36	1.60	1.67	200
22	24	-7.24	-3.61	0.47	2.50	8.09	200
23	24	-2.37	-1.63	3.64	2.39	2.88	200
24	25	-0.48	-2.52	-3.23	0.01	2.56	200

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
25	26	-3.69	-2.47	13.17	0.74	4.44	200
25	27	3.22	-0.03	3.90	-1.84	3.22	200
27	28	15.52	-19.94	-15.52	17.43	25.27	200
27	29	-6.45	-1.76	6.35	1.57	6.68	200
27	30	-7.39	-1.76	7.21	1.41	7.60	200
29	30	-3.86	-0.64	3.82	0.57	3.91	200

Case –2: Real and Reactive Power Cost Minimization with optimized tap settings and optimized Reactive support at load buses

Table B1.6: LSF = 1.08

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.012	-0.173	**LSF = 1.08**
2	1.034	-3.180	2.176	0.112	
3	1.031	-5.128	2.285	0.329	
4	1.026	-6.277	2.375	0.439	
5	1.006	-10.138	2.539	0.550	
6	1.022	-7.322	2.441	0.545	
7	1.007	-9.044	2.511	0.562	
8	1.023	-7.367	2.482	0.681	
9	1.046	-9.255	2.466	0.632	
10	1.035	-11.308	2.508	0.751	
11	1.091	-7.383	2.430	0.494	
12	1.050	-10.535	2.519	1.000	
13	1.088	-9.348	2.469	1.396	
14	1.033	-11.520	2.600	0.993	
15	1.028	-11.618	2.603	0.973	
16	1.035	-11.147	2.537	0.914	
17	1.030	-11.496	2.539	0.813	
18	1.017	-12.274	2.636	0.919	
19	1.014	-12.453	2.637	0.881	
20	1.019	-12.230	2.607	0.850	
21	1.021	-11.822	2.560	0.789	
22	1.022	-11.814	2.560	0.790	
23	1.015	-12.080	2.634	0.931	
24	1.007	-12.319	2.636	0.853	
25	0.998	-12.131	2.604	0.798	
26	0.978	-12.603	2.682	0.850	
27	1.002	-11.727	2.552	0.738	
28	1.019	-7.767	2.473	0.595	
29	0.980	-13.117	2.675	0.773	
30	0.967	-14.118	2.760	0.787	

Objective function:

Minimize Cost of Real Power

Value of objective function = 7.3974

Total Line Losses:

Real power loss = 0.091761 pu
Reactive power loss = 0.450014 pu

Table B1.7: Transformer Tap Settings

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.980
4	12	0.9	1.1	0.932	0.900
28	27	0.9	1.1	0.968	1.100

Table B1.8: Optimized Capacitor Placement at load buses.

Bus Number	Capacitance (pu MVAR)
12	0.052

Table B1.9: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.623	-0.086	
2	0.586	0.056	
5	0.246	0.268	
8	0.350	0.329	
11	0.179	0.242	
13	0.169	0.300	

Table B1.10: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-95.56	2.82	93.97	-7.49	95.60	200
1	3	-54.34	3.20	53.13	-7.55	54.43	200
2	4	-29.78	6.80	29.30	-8.25	30.55	200
2	5	-60.96	-1.31	50.31	-5.37	60.97	200
2	6	-39.22	7.09	59.31	-5.49	39.85	200
3	4	-50.63	4.45	38.37	-9.61	50.83	200
4	6	-42.32	2.29	42.14	-2.91	42.38	200
4	12	-32.66	-41.70	-14.19	2.75	52.97	200
5	7	14.10	-3.00	37.91	4.79	14.41	200
6	7	-38.28	-5.91	0.11	-0.93	38.73	200
6	8	-0.08	1.01	18.06	44.91	1.01	200
6	9	-18.06	-49.34	13.44	9.21	52.54	200
6	10	-13.44	-10.59	34.18	8.65	17.11	200
6	28	-13.18	-0.78	-17.93	-23.72	13.21	200
8	28	-3.91	1.83	32.66	36.19	4.31	200
9	10	-34.18	-9.93	-16.91	-30.00	35.60	200
9	11	17.93	22.17	8.19	2.41	28.52	200
10	17	-5.63	-4.46	18.68	6.95	7.18	200
10	20	-9.51	-3.83	1.73	0.74	10.25	200
10	21	-17.20	-10.71	7.39	3.52	20.26	200
10	22	-8.44	-4.97	5.62	4.42	9.79	200
12	13	16.91	28.60	3.74	1.62	33.22	200
12	14	-8.27	-2.58	6.10	1.62	8.67	200
12	15	-18.93	-7.43	2.77	0.68	20.34	200
12	16	-7.45	-3.64	9.42	3.63	8.29	200
14	15	-1.74	-0.74	-7.13	-2.90	1.89	200
15	18	-6.14	-1.71	17.07	10.43	6.38	200
15	23	-5.74	-3.37	8.37	4.84	6.66	200
16	17	-3.75	-1.65	-1.13	-1.22	4.10	200
18	19	-2.78	-0.69	5.70	3.29	2.86	200
19	20	7.11	2.86	7.17	3.50	7.66	200
21	22	1.13	1.22	2.36	1.60	1.67	200
22	24	-7.24	-3.61	0.47	2.50	8.09	200
23	24	-2.37	-1.63	3.64	2.39	2.88	200
24	25	-0.48	-2.52	-3.23	0.01	2.56	200

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
25	26	-3.69	-2.47	13.17	0.74	4.44	200
25	27	3.22	-0.03	3.90	-1.84	3.22	200
27	28	15.52	-19.94	-15.52	17.43	25.27	200
27	29	-6.45	-1.76	6.35	1.57	6.68	200
27	30	-7.39	-1.76	7.21	1.41	7.60	200
29	30	-3.86	-0.64	3.82	0.57	3.91	200

Case –3: Real and Reactive Power Cost Minimization with optimized tap settings and optimized Reactive support at load buses

Table B1.12: LSF = 1.12

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.013	-0.226	**LSF = 1.12**
2	1.034	-3.461	2.221	0.170	
3	1.030	-5.481	2.357	0.458	
4	1.026	-6.714	2.473	0.606	
5	1.006	-10.794	2.623	0.607	
6	1.022	-7.841	2.581	0.807	
7	1.006	-9.640	2.641	0.743	
8	1.023	-7.921	2.682	1.141	
9	1.046	-9.887	2.622	0.756	
10	1.035	-12.000	2.675	0.870	
11	1.091	-8.015	2.581	0.496	
12	1.050	-11.199	2.632	1.000	
13	1.088	-10.012	2.586	1.232	
14	1.036	-12.296	2.723	1.000	
15	1.029	-12.377	2.740	1.000	
16	1.035	-11.834	2.680	0.965	
17	1.029	-12.195	2.698	0.913	
18	1.018	-13.037	2.791	0.981	
19	1.014	-13.211	2.800	0.963	
20	1.019	-12.973	2.772	0.941	
21	1.021	-12.538	2.733	0.912	
22	1.022	-12.531	2.732	0.913	
23	1.016	-12.850	2.790	0.999	
24	1.008	-13.090	2.815	0.975	
25	1.004	-12.959	2.807	0.975	
26	0.989	-13.664	2.895	1.000	
27	1.007	-12.484	2.763	0.963	
28	1.019	-8.328	2.632	0.894	
29	0.986	-13.967	2.908	0.994	
30	0.975	-15.056	3.009	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 7.6114

Total Line Losses:

Real power loss = 0.10206 pu
Reactive power loss = 0.479253 pu

Table B1.13: Transformer Tap Settings

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.979
4	12	0.9	1.1	0.932	0.900
28	27	0.9	1.1	0.968	0.986

Table B1.14: Optimized Capacitor Placement at load buses

Bus Number	Capacitance (pu MVAR)
12	0.033
14	0.013
15	0.019
26	0.015
30	0.011

Table B1.15: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.747	-0.113	
2	0.586	0.085	
5	0.246	0.295	
8	0.350	0.350	
11	0.179	0.243	
13	0.169	0.300	

Table B1.16: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-104.03	5.15	102.14	-10.71	104.16	200
1	3	-58.26	3.47	56.87	-8.48	58.36	200
2	4	-31.42	6.78	30.89	-8.40	32.15	200
2	5	-64.29	-0.92	53.91	-6.49	64.30	200
2	6	-41.55	7.31	62.46	-6.64	42.19	200
3	4	-54.27	5.44	40.60	-10.16	54.55	200
4	6	-45.33	3.45	45.10	-4.26	45.46	200
4	12	-34.73	-41.43	-14.82	3.63	54.06	200
5	7	14.71	-3.91	39.45	4.35	15.22	200
6	7	-39.86	-5.60	1.30	-2.84	40.26	200
6	8	-1.30	2.84	17.69	-1.74	3.12	200
6	9	-17.69	1.14	14.71	18.21	17.73	200
6	10	-14.71	-21.05	35.27	9.20	25.68	200
6	28	-13.95	-0.84	-17.93	-24.24	13.98	200
8	28	-3.90	1.43	34.73	35.68	4.15	200
9	10	-35.27	-10.56	-16.91	-30.00	36.82	200
9	11	17.93	22.66	8.52	2.55	28.89	200
10	17	-5.83	-4.39	19.42	7.43	7.30	200
10	20	-9.85	-3.86	1.82	0.82	10.58	200
10	21	-17.83	-11.11	7.69	3.89	21.01	200
10	22	-8.74	-5.15	5.81	4.35	10.14	200
12	13	16.91	28.60	3.91	1.91	33.22	200
12	14	-8.61	-2.74	6.37	1.81	9.04	200
12	15	-19.70	-7.96	2.91	0.83	21.25	200
12	16	-7.76	-4.03	9.75	3.64	8.74	200
14	15	-1.82	-0.83	-7.38	-2.89	2.00	200
15	18	-6.41	-1.90	17.68	10.80	6.69	200
15	23	-5.97	-3.65	8.67	5.01	7.00	200
16	17	-3.91	-1.95	-1.22	-1.29	4.37	200
18	19	-2.91	-0.84	5.93	3.55	3.03	200
19	20	7.35	2.85	7.37	3.59	7.89	200
21	22	1.22	1.29	2.46	1.80	1.78	200
22	24	-7.45	-3.71	0.42	2.49	8.32	200
23	24	-2.47	-1.83	3.78	2.48	3.07	200
24	25	-0.44	-2.52	-3.42	-0.09	2.55	200

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
25	26	-3.83	-2.57	13.92	0.73	4.61	200
25	27	3.41	0.07	3.89	-1.44	3.41	200
27	28	16.19	-19.60	-16.19	17.05	25.43	200
27	29	-6.70	-1.84	6.60	1.64	6.95	200
27	30	-7.68	-1.84	7.48	1.46	7.90	200
29	30	-4.01	-0.67	3.96	0.59	4.06	200

Case –4: **Real and Reactive Power Cost Minimization with optimized tap settings and optimized Reactive support at load buses**

Table B1.17: LSF = 1.16

Bus Number	Voltage (pu)	Angle (Degrees)	Bus Marginal Price (Real)	Bus Marginal Price (Reactive)	Remarks
1	1.050	0.000	2.014	-0.282	**LSF = 1.16**
2	1.034	-3.743	2.262	0.220	
3	1.030	-5.838	2.400	0.502	
4	1.025	-7.155	2.532	0.671	
5	1.006	-11.452	2.700	0.660	
6	1.022	-8.372	2.660	0.908	
7	1.006	-10.242	2.724	0.826	
8	1.023	-8.481	2.782	1.310	
9	1.046	-10.515	2.709	0.799	
10	1.037	-12.674	2.766	0.911	
11	1.091	-8.644	2.667	0.487	
12	1.050	-11.826	2.702	1.000	
13	1.088	-10.640	2.657	1.167	
14	1.037	-12.996	2.799	1.000	
15	1.032	-13.125	2.821	1.000	
16	1.035	-12.496	2.762	0.983	
17	1.030	-12.873	2.788	0.948	
18	1.020	-13.784	2.880	0.997	
19	1.016	-13.951	2.894	0.988	
20	1.020	-13.698	2.866	0.971	
21	1.023	-13.240	2.828	0.952	
22	1.024	-13.237	2.827	0.952	
23	1.023	-13.684	2.879	1.000	
24	1.015	-13.889	2.912	1.000	
25	1.018	-13.834	2.911	1.000	
26	1.008	-14.704	3.001	1.000	
27	1.024	-13.323	2.868	1.000	
28	1.021	-8.932	2.718	1.000	
29	1.010	-14.991	3.018	1.000	
30	1.000	-16.101	3.119	1.000	

Objective function:

Minimize Cost of Real Power

Value of objective function = 7.9746

Total Line Losses:

Real power loss = 0.112908 pu

Reactive power loss = 0.519414 pu

Table B1.18: Transformer Tap Settings

From Bus	To Bus	Tap Limits		Specified Tap Setting	Optimized Tap Setting
		Tap Upper Limit (p.u)	Tap Lower Limit (p.u)		
6	9	0.9	1.1	0.978	0.900
6	10	0.9	1.1	0.969	0.977
4	12	0.9	1.1	0.932	0.900
28	27	0.9	1.1	0.968	1.002

Table B1.19: Optimized Capacitor Placement at load buses

Bus Number	Capacitance (pu MVAR)
12	0.002
14	0.017
15	0.035
23	0.019
25	4.21 E -4
26	0.027
27	0.021
28	0.003
29	0.013
30	0.027

Table B1.20: Generation Details

Bus Number	Real Power Generation (pu)	Reactive Power Generation (pu)	Remarks
1	1.871	-0.140	
2	0.586	0.110	
5	0.246	0.319	
8	0.350	0.350	
11	0.179	0.239	
13	0.169	0.300	

Table B1.21: Line Flows

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
1	2	-112.50	7.45	110.29	-13.97	112.74	200
1	3	-62.16	3.83	60.57	-9.54	62.27	200
2	4	-33.03	6.89	32.44	-8.67	33.75	200
2	5	-67.62	-0.56	57.46	-7.75	67.62	200
2	6	-43.89	7.73	65.60	-7.82	44.56	200
3	4	-57.88	6.55	42.82	-10.91	58.25	200
4	6	-48.49	4.96	48.22	-5.89	48.75	200
4	12	-36.55	-41.28	-15.47	4.33	55.14	200
5	7	15.34	-4.64	41.00	4.09	16.03	200
6	7	-41.45	-5.44	2.52	-3.89	41.81	200
6	8	-2.53	3.89	18.75	-1.11	4.64	200
6	9	-18.75	0.44	15.34	18.11	18.75	200
6	10	-15.34	-21.04	36.28	9.08	26.04	200
6	28	-14.91	0.31	-17.93	-24.32	14.92	200
8	28	-3.92	1.55	36.55	35.28	4.22	200
9	10	-36.28	-10.52	-16.91	-30.00	37.78	200
9	11	17.93	22.72	8.86	1.50	28.94	200
10	17	-6.09	-4.65	20.03	5.85	7.66	200
10	20	-10.22	-3.69	1.90	1.04	10.87	200
10	21	-18.33	-10.94	7.94	3.94	21.35	200
10	22	-8.95	-4.96	6.07	4.60	10.23	200
12	13	16.91	28.60	4.01	1.89	33.22	200
12	14	-8.95	-1.69	6.60	2.20	9.11	200
12	15	-20.30	-6.39	3.01	1.18	21.28	200
12	16	-8.01	-4.09	10.12	3.46	8.99	200
14	15	-1.91	-1.04	-7.65	-2.68	2.18	200
15	18	-6.65	-2.30	18.18	10.62	7.04	200
15	23	-6.10	-3.69	8.88	4.81	7.13	200
16	17	-4.02	-1.93	-1.42	-1.93	4.46	200
18	19	-3.02	-1.19	6.05	3.59	3.24	200
19	20	7.63	2.63	7.39	2.77	8.07	200
21	22	1.42	1.93	2.46	1.77	2.39	200
22	24	-7.46	-2.88	0.10	1.41	8.00	200
23	24	-2.47	-1.80	3.92	1.05	3.05	200
24	25	-0.11	-1.41	-3.88	0.26	1.42	200

From Bus	To Bus	From Bus Injection		To Bus Injection		Line Flow (MVA)	Line Limit (MVA)
		Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)		
25	26	-3.96	-1.12	14.88	-0.43	4.12	200
25	27	3.86	-0.29	3.92	-1.57	3.87	200
27	28	19.05	5.97	-19.05	-7.53	19.97	200
27	29	-6.95	-1.43	6.84	1.23	7.10	200
27	30	-7.97	-1.23	7.76	0.84	8.06	200
29	30	-4.15	-0.22	4.11	0.14	4.16	200